COMPARATIVE STUDY OF STRUCTURAL BEHAVIOURS AND DESIGN OF LONG CYLINDRICAL CONCRETE SHELLS USING BEAM AND CLASSICAL METHODS

A Thesis Submitted
in partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By

PRABIR CHANDRA BASU

to the

DEPARTMENT OF CIVIL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

SEPTEMBER 1975

to my parents

CERTIFICATE

This is to certify that the thesis entitled, 'COMPARATIVE STUDY OF STRUCTURAL BEHAVIOURS AND DESIGN OF LONG CYLINDRICAL CONCRETE SHELLS USING BEAM AND CLASSICAL METHODS' , by Mr. Prabir Chandra Basu was carried out under my guidance and has not been submitted elsewhere for a degree.

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Prabir C Basu

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NOTATIONS

Greek Letters

α, α'	=	Roots of characteristic eqns.
β, β'	=	Roots of characteristic eqns.
γ	=	Density of concrete, const. for roots of characteristic eqns.
η	=	Const. for roots of characteristic eqns.
λ	=	Const. for the solution of stress resultant.
ϕ_k	=	Semicentral angle
ϕ	=	Angle at any circumpherial section
∇	=	Differential opperator
ϕ'	=	$\phi_k - \phi$
σ_x	=	Longitudinal stress
τ	=	Shear stress
Ψ_1 to Ψ_2	=	Trigonometrical and hyperbolic multiples
θ	=	Rotation
δ	=	Ratio of the deformations
ϵ	=	Error in σ_x

Capital Letters

A	=	Area of cross section
$[A]$	=	Co-eff. matrix for arch analysis
A_m	=	Const. for homogeneous solution
B_1, B_2, B_3, B_4	=	Shell const. for homogeneous solution
$\{C\}$	=	Constant vector for arch analysis
C_b	=	Beam constant or const. for perticular integral
E	=	Modulus of elasticity

F	=	Nodal forces
G	=	Coeff. of Torsional rigidity
H	=	Harizontal end reaction
I	=	Moment of Inertia
J	=	Polar moment of Inertia
$K_{11}, K_{21}, K_{22}, K_{31}$	=	Consts for arch analysis
L	=	Span
L_c	=	Chord length
M_m	=	End moment for shell or bending moment
M	=	Transverse moment for shell or bending moment
$M_{\phi x}, M_{x\phi}$	=	Torsional moment for shell
M_x	=	Longitudinal moment for shell
N	=	Axial thurst for arch
N_ϕ	=	Inplane force of shell
N_1, N_2	=	Principal force
N_x	=	Longitudinal force for shell
$N_{\phi x}, N_{x\phi}$	=	Shear force for shell
P_e	=	Eff. prestressing force
Q	=	First moment of area or const.for solutionof characterestic equationof shell
Q_x	=	Radial shear
Q_ϕ	=	Shear in the ϕ direction
R	=	Radius
S_f	=	Shear force
S	=	End correction force, (Shear)
T	=	Const., Radius to thickness ratio

U	=	Strain energy
V	=	Vertical correction force
X,Y,Z	=	Co-ordinate system
Z	=	Distance of C.G.
Z_y	=	Distance of fiber
Z_r	=	Distance of the C.G. of the element from the whole section C.G.

Small Letters:

a	=	Width of arch, const.of homogeneous solution of shell eqn.
b	=	Width of edgebeam, const. of homogeneous solution of shell eqn.
c	=	Coeff. of deformations, stress resultants, const. of homogeneous solutionof shell eqns, Consts.
d	=	depth. of edge beam, const. of homogeneous of shell eqn.
ds	=	Elementery arc length
dy	=	Horizontal projection of ds
dz	=	Vertical projection of ds,
e	=	Eccentricity of tendon for beam theory
e_e	=	End eccentricity of tendon for beam theory
e_o	=	End eccentricity of tendon in edge beam
e_m	=	Mid eccentricity of tendon in edge beam
e_h	=	Horizontal eccentricity of the shell central line from the centre line of edge beam
e_v	=	Vertical eccentricity of centre line of shell from centre line of edge beam
g	=	Sag
h	=	Total depth of trnsverse section of the shell with edge beam

i	=	Nodal point, $\sqrt{-1}$
k	=	Constants, $n\pi x/l$, no. of slices of arch analysis
k_m, k_n	=	Const. for arch analysis
l	=	Span
l_c	=	Chord length
m	=	Root of the charecteristic eqn., $n\pi x/l$, total no. of nodes.
n	=	No. of fourier terms, $k-1$
p	=	Nondimenerionalised coeff. for prestressing force, external loading
q	=	Intensity of loading on shell
t	=	Thickness of shell
u	=	Displacements
w	=	Deformations

Subscripts

1	=	For N_x when used in first place, for V when used in second place
2	=	For N_x when used in first place, for S when used in second place
3	=	For N_x when used in first place, for H when used in second place
4	=	For N_x when used in first place, for M when used in second place
b	=	Edge beam
c	=	Shell without edge beam
e	=	Effective
f	=	force

g	=	Load on the surface area of shell
h	=	Horizontal direction, due to H
i	=	Nodal points,
k	=	Shell edge
l	=	Line load
m	=	Due to rotation, membrane action
r	=	Radial direction
s	=	Shear force, shell with edge beam
u	=	Uniformly distributed load on chord length
v	=	Vertical direction
w	=	Due to deformation
x,y,z	=	In the direction of co-ordinate axes
I	=	Const. for moment of inertia
H	=	Due to horizontal force
M	=	Due to moment
V	=	Due to vertical force
Z	=	Const. for C.G.
ϕ	=	In the direction of ϕ.

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SYNOPSIS

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The thesis presents the comparative study of the structural behaviour and design of long cylindrical concrete shell having span to radius ratio greater than 3 , are assumed as long shells and such shells with edge beam are considered for the investigation. Both reinforced and prestressed concrete shells are studied. Only edge beams of prestressed shells were provided with prestressing force. The response of the stress resultants and deformations are taken as structural behaviour parameters.

Paramatric study was made to investigate the influence of different shell parameters and prestressing on the behaviour evaluated by both the theories and to find out the domain of applicability of the beam theory. It was found that in case of reinforced concrete shell, beam theory is applicable for all

normal values of radius to thickness and depth of edge beam to span ratios, semicentral angle and span to radius ratio greater than 3. , within 4 to 15 percent of error.

Prestressing reduces the stress resultants and deformations with a suitable combination of depth of edge beam, and ~~wid~~eccentricities of the tendon. The discrepency in the assumptions of beam theory increases with the increase of depth of edge beam and prestressing force. Restraints against the horizontal and rotational deformations yield the stress resultant which are in nature to compensate each other. No shell with coefficient of prestressing more or equal to 0.4 and depth of edge beam to span ratio more or equal to 0.04 could be designed by beam theory.

It was found that the arch analysis by column analogy method considering a thin strip of the shell with edge beam is not efficient. A simplified formulation for arch analysis based on strain energy method, satisfying the compatibility of deformation at the interface of the shell and edge beam, is developed. With the help of this arch analysis a systematic and fairly simple method, detailed in six steps, is presented. The proposed method gives good results for single barrel shell.

A series of table is given to make the computational work easy for the proposed method.

CHAPTER-I

INTRODUCTION

1.1 GENERAL:

Shells are one of the economical, effective and elegant structural system for the purpose of roofing of a large unobstructed area. They tend to carry loads primarily by direct stress acting in their plane, because of their geometry and small flexural rigidity of skin which catagorise them to stressed skin structures. Among the different types of shell, cyllindrical shell is most popular for roofing purpose. They have extensive application in constructing airport hangers, industrial buildings, auditoriums, exhibition stalls, godowns, markets, garage units, bus terminus etc.

For small span, shells are not provided with edge beam, but as span increases edge beams are provided to reduce the stresses. For very long span prestressing of edge beam yields the optimal utilization of structural properties of cylindrical shell roofs. Prestressing of edge beam has certain advantages as it reduces stress resultant and hence offers greater safety against buckling; deflections; consumptions of steel and dead weight of the structures. With the suitable selection of end and mid eccentricity and

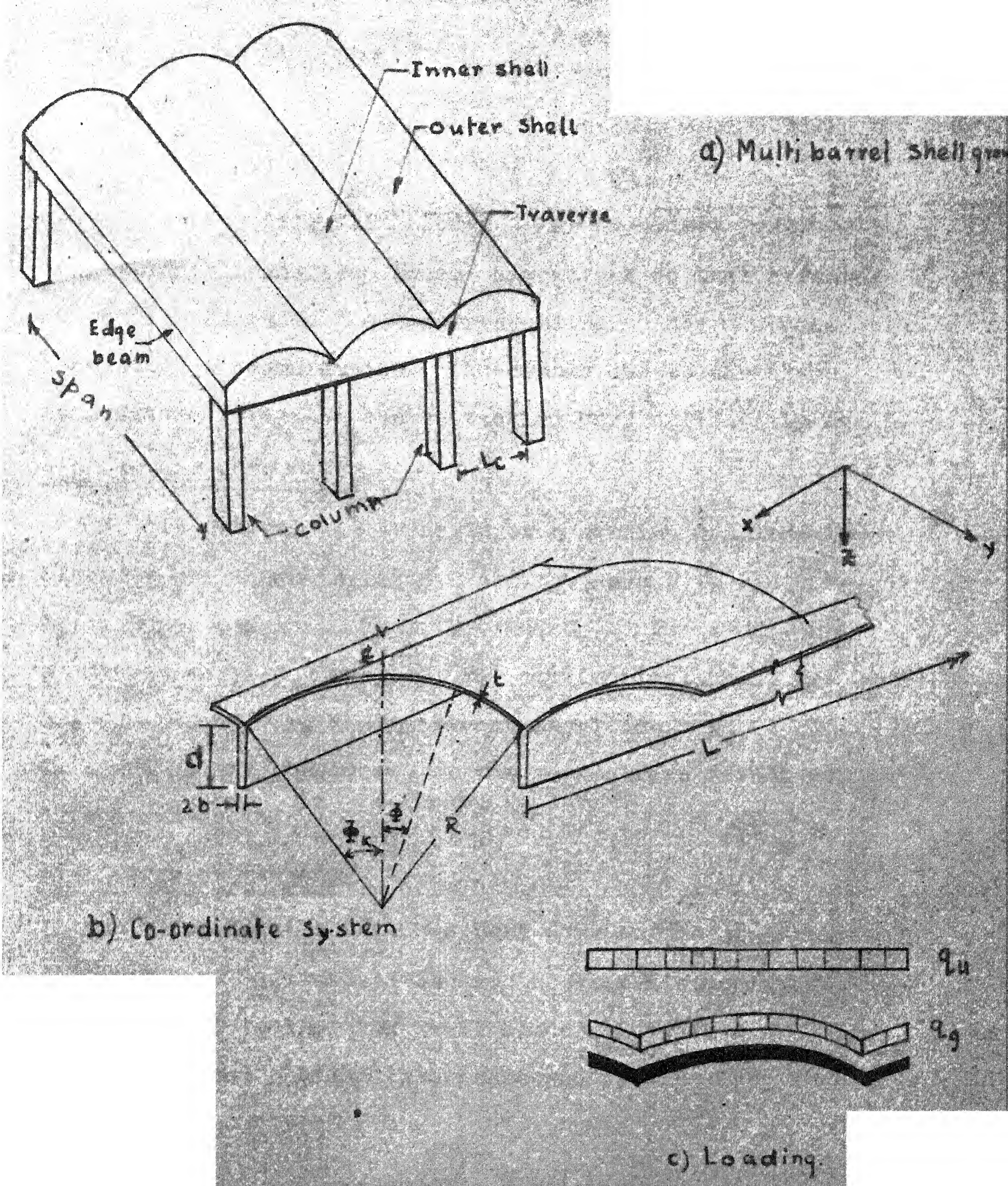


Fig. 1.1 MULTIPLE CYLINDRICAL SHELL ROOF

magnitude of prestressing force it is sometimes possible to eleminate transverse moment altogether so that stresses in shell reduces to those corresponding to the membrane plus a constant compression. The cracks due to shrinkage, temperature variation can be tackled very efficiently by it.

1.2 STRESS RESULTANTS:

In general the stresses of a continuum structural elements are shown in Fig. 12 and they are N_x , $N_{x\phi}$, $N_{\phi x}$, M_ϕ , $M_{\phi x}$, $M_{x\phi}$, M_x , Q_x and Q_ϕ . For a uni-axi-symmetric shell $N_{x\phi} = N_{\phi x}$. For cylindrical shell $M_{x\phi} = M_{\phi x}$ Q_x, Q_ϕ , have very insignificant magnitude. N_x , $N_{x\phi}$, N_ϕ , and M_ϕ are considered as the main stress resultants in this investigation.

1.3 LITERATURE REVIEW:

Considerable work had been done on the analysis and design of Cylindrical Shell. However discussion here is restricted to the work available on analysis and design of cylindrical shell with prestressed edge beam by classical and 'Beam Theory'.

In 1949 H. Lundgren(3) first introduced a simple theory to analyse long cylindrical shell by 'Beam Theory'. He compared the transverse section of a cylindrical shell as a curved section of a beam and analyse the shell for N_x and

$N_{x\phi}$ considering the shell as a beam spanning from one edge transverse to another in longitudinal direction. For the calculation of M_{ϕ} and N_{ϕ} he performed the arch analysis considering a unit width strip of shell as an arch loaded by the external loading and $N_{x\phi}$ which is calculated by beam calculation. The arch calculation that he did was for the interior shell and analysed it by column analogy method. J. Chinn (12) in 1959 demonstrated that the interior shell of a group of shells without edge beam even with L/R $\geqslant$ 1.11 can be analysed by beam theory. For arch calculation he applied the elastic centre method. He also compared stress resultants obtained by beam theory and ASCE Manual (31). In the same year A.L Parme and H.W. Conner (14) introduced tables for the correction of stress resultants obtained by beam theory. Ramaswami (7) described almost all of the so called exact theory to analyse cylindrical shell. He also demonstrated the procedure of designing the cylindrical shell with prestressed edge beam and analysed an interior shell of a group of shell without edge beam having L/R=2 using column analogy method for arch calculation. Gibson(1) in 1968 presented the method to analyse the cyllindrical shell by assuming it as a shallow shell. A detailed method to analyse a shell with prestressed edge beam was put by him and a comparison of N_x obtained by exact method and beam

theory for shell without edge beam and with prestressed and non prestressed edge beam had been done by him. Manual No. 31 of ASCE on the Design of Shell Roofs(27) contains a series of comprehensive tables and charts which reduces the lot of computational hazard. P. Dayaratnam(8) illustrated completely the procedure to design a single barrel shell with a prestressed edge beam using ASCE Manual No. 31. D.P. Billington(2) discussed the classical shell theory by theory of shallow shell and use of ASCE Manual-31 and simplified the design problem. He also demonstrated the reason of non agreement of stress resultant obtained by exact solution and beam theory for a single barrel shell without edge beam of L/R=2.5 and introduced a chart for stress resultant prepared by beam arch approximation. Marshall(9) had shown it is sometimes possible to eliminate transverse moment altogether so that the stresses in the shell area reduced to those corresponding a membrane plus a constant compression. Ramaswami and Saeed(34) used Beam Method to analyse a Hyperboloid paraboloid shell.

Very little work is available on the structural behaviour of the cylindrical shells with edge beams by beam theory for reinforced or prestressed shell.

1.4 PROPOSED INVESTIGATION:

Design of cylindrical shell by so called classical method is complicated and involves tedious computations, which, as a result, is not an attractive procedure to the engineers in practice. Beam theory, sufficiently accurate for reinforced concrete cylindrical shell, involves less complicacy. Though the method described in ASCE Manual-31 is not that complicated as other classical methods but it is not suitable for a shell with any type of combination of L/R , R/t and ϕ_K. Again prestressing is desirable for optimal utilization of stressed skin structures. So there is a need for research on beam theory to explore its applicability to analylyze the stress resultant of long cylindrical shell with prestressed edge beam.

The summary of proposed investigation is:

1. Analysis of long cylindrical shell with edge beam by classical and beam theories. Then make a comparative study of the structural behaviour.
2. Parametric study of the structural behaviour of the shell by classical and beam theories.
3. Develop the formulations for arch analysis by strain energy method and illustrate the procedure.
4. Series of tables are given to design barrel shell roof by the above method.

1.5 CO-ORDINATE SYSTEMS AND SIGN CONVENTIONS:

The co-ordinate system adopted in the investigation is shown in the Figure 1.1(b). The following sign conventions are used unless otherwise mentioned (ref. fig. 2.2).

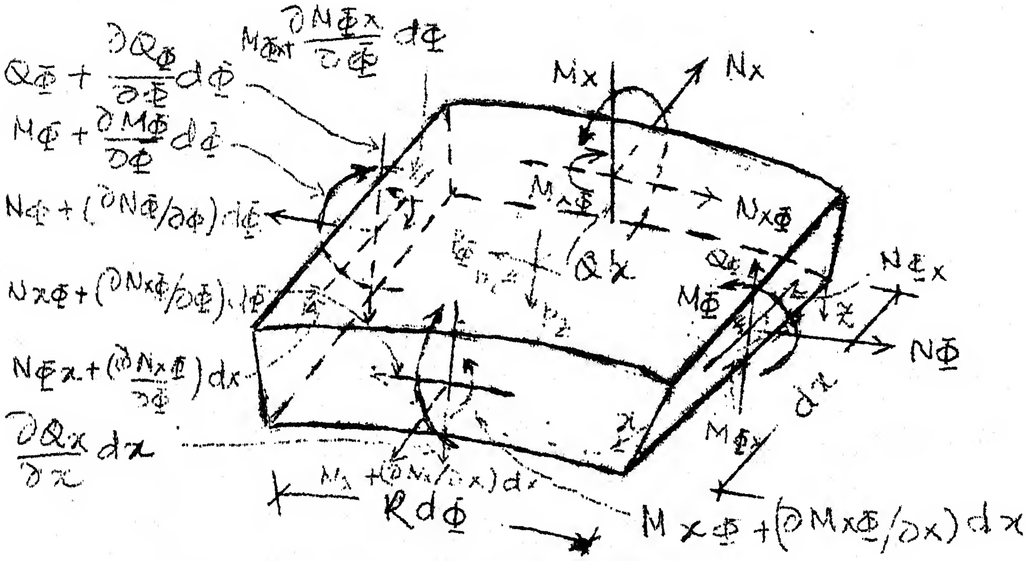


Fig. 1.2 STRESS RESULTANTS

N_x = Longitudinal force considered positive if it is tensile.

N_ϕ = Transverse inplaneforce in outward direction is positive.

$N_{x\phi}$ = Tangential shearing force is positive when is creates tension in the increasing direction of ϕ and x.

M_ϕ = Transverse moment is positive if it induces tension at the bottom fiber.

u_x = Longitudinal deformation is positive if it increases along positive x-direction.

u_y = It is positive if it increases along positive y-direction.

u_r = Radial displacement is positive if it acts outwards.

w_v = Vertical deformation in downward direction is positive.

w_h = Horizontal deformation in outward direction is positive.

w_x = Longitudinal deformation is positive when it is extensional in nature.

w_m = Rotational deformation is positive when it has anticlock wise sense.

For other purpose general sign convention will be followed. Throughout the study sign convention, as mentioned above, will be followed unless other-wise mentioned.

1.6 CLASSICAL SHELL THEORY(8):

Classical shell theory developed, has the following steps or system,

i) Primary system:- Shell resisting surface load solely by direct inplane stress resultant i.e. membranec state.

ii) Errors:- Forces generated by membrane theory at edges which does not satisfy the boundary conditions.

iii) Corrections:- Corrective line loads are applied at free edges to satisfy actual boundary conditions.

iv) Bending Solution:- These corrective line loads will cause bending of the shell for which bending solution is

required. Membrane solution is obtained considering, that, the shell is loaded at the edge by corrective line loads.

Complete solution yields by superimposing membrane solution, obtained considering shell is loaded by external loads, on bending solution, considering edge disturbance as the loading on the shell.

a) Assumptions:

1. Materials are homogeneous, isotropic and obey Hooke's-law.

2. Deformation of shell produced due to loads are small.

3. Thickness of shell is small when compared with is is other dimensions and radii of curvature.

4. Kirchoff's hypothesis holds good i.e.,

i) Rectilinear elements normal to the middle surface of the shell remain rectilinear and normal even after deformation and do not change their lengths.

ii) Normal stresses acting on the planes parallel to the middle surface are neglected in comparison with other stresses.

5. Poisson's ratio of concrete is neglected.

6. Shell is assumed to be simply supported spanning between two longitudinal traverses.

b) Membrane Theory:

Membrane theory assumes external loading and the surface of this shell are continuous.

The equilibrium equations for membrane state of stress (ref. Fig. 1.2)) are as follows:

$$\frac{\partial N_x}{\partial x} + \frac{1}{R} \frac{\partial N_{x\phi}}{\partial \phi} + q_x R = 0 \qquad (1.1)$$

$$\frac{1}{R} \frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + q_y R = 0 \qquad (1.2)$$

$$N_\phi + q_z R = 0 \qquad (1.3)$$

Membrane solution is obtained by integrating above solution which are given below,

i) For loading acting on the chord area of the shell (q_u)

$$N_x = - q_u R \left(\frac{1}{R} \right)^2 (\cos^2 \phi' - \sin^2 \phi') \frac{3}{n^2 \pi^2} \sin \frac{n \pi x}{l} \qquad (1.4)$$

$$N_{x\phi} = - q_u R \left(\frac{1}{R} \right) \cos \phi' \sin \phi' \frac{3}{n\pi} \cos \frac{n\pi x}{l} \qquad (1.5)$$

$$N_\phi = - q_u R \cos^2 \phi' \frac{1}{n} \sin \frac{n\pi x}{l} \qquad (1.6)$$

ii) For the loading action on the surface of the shell (q_g)

$$N_x = - q_g R \left(\frac{1}{R}\right)^2 \cos \phi' \frac{2}{n^2\pi^2} \sin \frac{n\pi x}{l} \quad (1.7)$$

$$N_{x\phi} = - q_g R \left(\frac{1}{R}\right) \sin \phi' \frac{2}{n\pi} \cos \frac{n\pi x}{l} \quad (1.8)$$

$$N_\phi = - q_g R \cos \phi' \frac{1}{n} \sin \frac{n\pi x}{l} \quad (1.9)$$

The expression of membrane displacements are given below,

i) For q_u,

$$u_x = q_u R \frac{R}{Et}\left(\frac{1}{R}\right)^3 (\cos^2 \phi' - \sin^2 \phi') \frac{3}{n^3\pi^3} \cos \frac{n\pi x}{l} \quad (1.10)$$

$$u_y = - q_u R \frac{R}{Et} \left(\frac{1}{R}\right)^4 \cos \phi' \sin \phi \left[2 + \left(\frac{n\pi R}{l}\right)^2\right] \sin \frac{n\pi x}{l} \quad (1.11)$$

$$u_r = - q_u R \frac{R}{Et}\left(\frac{1}{R}\right)^4 \left\{ (\cos^2\phi' - \sin^2 \phi') \left[2 + \left(\frac{n\pi R}{l}\right)^2\right] + \frac{1}{6}\left(\frac{n\pi R}{l}\right)^4 \cos^2 \phi' \right\} \frac{6}{n^4\pi^4} \sin \frac{n\pi x}{l} \quad (1.12)$$

ii) For q_g ,

$$u = q_g R \frac{R}{Et} \left(\frac{1}{R}\right)^3 \text{Cos } \phi' \frac{2}{n^3\pi^3} \text{ Sin } \frac{n\pi x}{l} \qquad (1.13)$$

$$u_x = - q_g R \frac{R}{Et} \left(\frac{l}{R}\right)^4 \left(1+2n^2\pi^2 \left(\frac{R}{l}\right)^2\right) \text{Sin } \phi' \frac{2}{n^4\pi^4} \text{ Sin } \frac{n\pi x}{l} \qquad (1.14)$$

$$u_r = - q_g R \frac{R}{Et} \left(\frac{l}{R}\right)^4 \left(1+2n^2\pi^2\left(\frac{R}{l}\right)^2 + \frac{n^4\pi^4}{2} \left(\frac{R}{l}\right)^4\right)$$

$$\times \text{ Cos } \phi' \frac{2}{n^4\pi^4} \text{ Sin } \frac{n\pi x}{l} \qquad (1.15)$$

where , $\phi' = \phi_x - \phi$

For the expression 1. 4 to 1.15 the sinusoidal variation of loading along the longitudinal direction is assumed.

c) <u>Bending Solution:</u>

The equilibrium equations for an element on the shell (Fig. 1.2), are as follows,

$$\frac{\partial N_x}{\partial x} R + \frac{\partial N_{x\phi}}{\partial \phi} + q_x R = 0 \qquad (1.16)$$

$$\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{x} R - Q_\phi + q_\phi R = 0 \qquad (1.17)$$

$$\frac{\partial Q_x}{\partial x}R + \frac{\partial Q_\phi}{\partial \phi} + N\phi + Q_x R = 0 \qquad (1.18)$$

$$- \frac{\partial M_\phi}{\partial \phi} + \frac{\partial M_{x\phi}}{\partial x} R + Q_\phi R = 0 \qquad (1.19)$$

$$- \frac{\partial M_x}{\partial x} R - \frac{\partial M_{\phi x}}{\partial \phi} + Q_x R = 0 \qquad (1.20)$$

In the above sets of equations total number of equations is 6 and number of unknowns is 9. To solve this statically indeterminate systems the other conditions are taken from stress-displacement relationship which are given below;

$$N_x = Et \frac{\partial u_x}{\partial x} \qquad (1.21)$$

$$N_\phi = Et \left(\frac{1}{R} \frac{\partial u_y}{\partial \phi} - \frac{u_r}{R} \right) \qquad (1.22)$$

$$N_{x\phi} = \frac{Et}{2} \left(\frac{\partial u_y}{\partial x} + \frac{1}{r} \frac{\partial u_x}{\partial \phi} \right) \qquad (1.23)$$

$$M_x = - \frac{Et^3}{12} \frac{\partial^2 u_r}{\partial x^2} \qquad (1.24)$$

$$M_\phi = - \frac{Et^3}{12} \left(\frac{1}{R^2} \frac{\partial u_y}{\partial \phi} + \frac{1}{R^2} \frac{\partial^2 u_r}{\partial \phi^2} \right) \qquad (1.25)$$

$$M_{x\phi} = \frac{Et^3}{12} \left(\frac{1}{2R} \frac{\partial u_y}{\partial x} + \frac{1}{r} \frac{\partial^2 u_r}{\partial x \partial \phi} \right) \qquad (1.26)$$

with the help of the equations 1.16 to 1.26 the final 8th order differential equation with respect to w_r obtained as,

$$\nabla^8 u_r + \frac{12}{t^2}\frac{\partial^4 u_r}{\partial x^4} = \frac{12}{Et^3} R^3 \nabla^4 q_z - \frac{2\partial^3 q_\phi}{\partial x^2 \partial \phi} - \frac{1}{R^3}\frac{\partial^3 q_\phi}{\partial \phi} + \frac{1}{R}\frac{\partial^3 q_x}{\partial x \partial \phi^2} \qquad (1.27)$$

$$\text{Where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial}{\partial \phi^2} \qquad (1.27)$$

Homogeneous solution of eqn.(1.27) has the form

$$u_r = \sum_{n=1,3}^{\infty} A_m e^{m\phi} \operatorname{Sin} kx, \quad \text{where } k = \frac{n\pi}{l} \qquad (1.28)$$

putting Eq. (1.28) on the homogeneous part of 1.27 the characteristics eqn. stands as follows;

$$R^2 \left(- k^2 + \frac{m^2}{R^2} \right)^4 + \frac{12}{t^2} k^4 = 0 \qquad (1.29)$$

Roots of the Eq. (1.29)

$$m_{1,4} = \pm (\alpha \pm i\beta) \qquad (1.30)$$

$$m_{5,8} = \pm (\alpha' + i\beta) \qquad (1.31)$$

where

$$\alpha = \eta \sqrt{\frac{\sqrt{(1+ \gamma)^2 + 1} + (1+ \gamma)}{2}} \qquad (1.32)$$

$$\beta = \eta \sqrt{\frac{\sqrt{(1+\gamma)^2 + 1} - (1+\gamma)}{2}} \qquad (1.33)$$

$$\alpha' = \eta \sqrt{\frac{\sqrt{(1-\gamma)^2 + 1} - (1-\gamma)}{2}} \qquad (1.34)$$

$$\beta' = \eta \sqrt{\frac{\sqrt{(1-\gamma)^2 + 1} + (1-\gamma)}{2}} \qquad (1.35)$$

$$\gamma = \left(\frac{kR}{\eta}\right)^2 \qquad (1.36)$$

$$\eta^8 = 3\,(kR)^4 \left(\frac{R}{t}\right)^2 \qquad (1.36)$$

Therefore,

$$\begin{aligned} u_r = \Big[& A_1 e^{(\alpha_1 + i\beta_1)\phi} + A_2 e^{(\alpha_1 - i\beta_1)\phi} \\ = & + A_3 e^{(\alpha_1' + i\beta_1')\phi} + A_4 e^{(\alpha_1' - i\beta_1')\phi} \\ & + A_5 e^{(-\alpha_1 + i\beta_i)\phi} + A_6 e^{(-\alpha_1 - i\beta_1)\phi} \qquad (1.37) \\ & + A_7 e^{(-\alpha_1' + i\beta_1')\phi} + A_8 e^{(-\alpha_1' - i\beta_1')\phi} \Big] \text{Sin } kx \end{aligned}$$

Expressing A_1 .... A_8 in the form : $a + ib$ and measuring the angle ϕ' from the crown the stress resultant and the displacement can be expressed by following expression,

$$\begin{aligned} F = & \lambda (a B_1 - bB_2)\Psi_1 - (a B_2 + bB_1)\Psi_2 + (cB_3 - dB_4)\Psi_3 \\ & - (cB_4 + dB_3)\Psi_4 \qquad (1.38) \end{aligned}$$

where,

$$\Psi_1 = \cos \beta \phi' \cosh \alpha \phi'$$

$$\Psi_2 = \sin \beta \phi' \sinh \alpha \phi'$$

$$\Psi_3 = \cos \beta' \phi' \cosh \alpha' \phi' \qquad (1.39)$$

$$\Psi_4 = \sin \beta' \sinh \alpha' \phi'$$

For different stress resultant and displacements, the shell coefficients are given in Table A.1.1 and A.1.2 in Appendix-1.

Particular solution:- The particular integral of Eq. 1.27 may be taken as

$$q_x = q \cos(\phi_k - \phi) \sin kx \qquad (1.40)$$

Substituting of the Eq. 1.40 in Eq. 1.27 gives

$$C = \frac{1}{Et} \frac{R^2 k^4 + 4k^2 + 2/R^2}{(R^2 t^2/12)(k^2 + 1/R^2)^4 + k^4} \qquad (1.41)$$

Where $q = \dfrac{4}{\pi} q_g$

If the loading is q_u then the particular solution is

$$q_x = q \sin kx, \quad \text{where } q = \frac{4}{\pi} q_u$$

1.7 BEAM THEORY:

Analysis of long cylindrical shell by beam theory has essentially two steps, i) beam analysis , and ii) arch analysis.

a) Assumptions:

Beam theory is based on the same assumptions as classical theory given in article 1.6(a), except that certain factors are assumed to be negligible and they are:

i) Relative displacements within each transverse cross sections.

ii) Longitudinal bending moments M_x and radial shear forces Q_x on the shell.

iii) Torsional moments ($M_{x\phi}$)

iv) Strain from inplane shearing force.

b) Beam Analysis:

The object of beam analysis is to calculate N_x, $N_{x\phi}$.

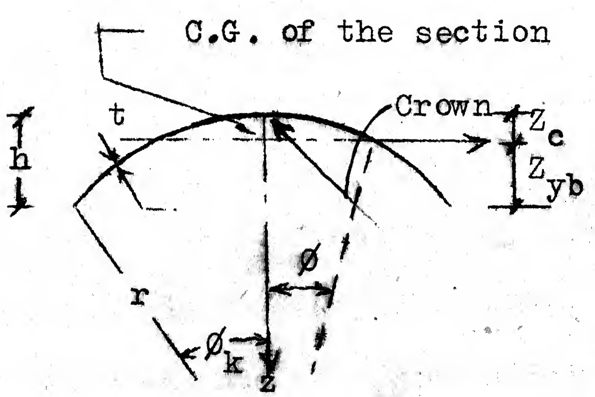


Fig. 1.3: Curved Beam Section

Area of the section is:

$$A_c = 2\,R\phi_k t \qquad (1.42)$$

Distance of the C.G. of the section from the crown is;

$$Z_c = (\,1 - \sin\phi_k / \phi_k)\,R \qquad (1.43)$$

Moment of inertia of the section is;

$$I_c = k_I \ R^3 t \quad , \quad k_I = \phi_k + \text{Sin}\ \phi_k\ (\text{Cos}\ \phi_k - 2\ \text{Sin}\ \phi_k/\phi_k\ ) \tag{1.44}$$

First moment of area at any section ϕ is;

$$Q = k_Q\ R^2 t \quad , \quad k_Q = (\ \text{Sin}\ \phi - \frac{\phi}{\phi_k} \text{Sin}\ \phi_k\ ) \tag{1.45}$$

Distance of the top fiber from C.G. = $Z_{yt} = Z_c$

Distance of the bottom fiber from C.G. = $Z_{yb} = h - Z_c$

Therefore

$$N_x \text{ at top (Comp.)} = q_o\ l^2\ \frac{t\ Z_{yt}}{I_c} \left[\frac{1}{2}\ \frac{x}{l}\ (1 - \frac{x}{l}\ ) \right] \tag{1.46}$$

$$N_x \text{ at bottom (tension)} = q_o\ l^2\ \frac{t\ Z_{yb}}{I_c} \left[\frac{1}{2}\ \frac{x}{l}\ (1 - \frac{x}{l}\ ) \right] \tag{1.47}$$

$$N_{x\phi} \text{ at any section } = q_o\ l\ \frac{Q}{2I}\ (\frac{1}{2} - \frac{x}{l}) \tag{1.48}$$

c) Arch Analysis:

Referen

Object of the arch analysis is to calculate N_ϕ , M_ϕ and Q_ϕ.

An elementary arch strip, which is under the action of q_u, q_g and specific shear, is considered. Eq.(1.45)express the magnitude of $N_{x\phi}$ at a section ϕ along x.

Therefore, $$\frac{\partial N_{x\phi}}{\partial x} = -\frac{q_o Q}{2 I} \tag{1.49}$$

So the specific shear at any longitudinal section is constant and its vertical component will act upwards and horizontal component inwards. The arch is divided into a no. of nodes of arc length ds.

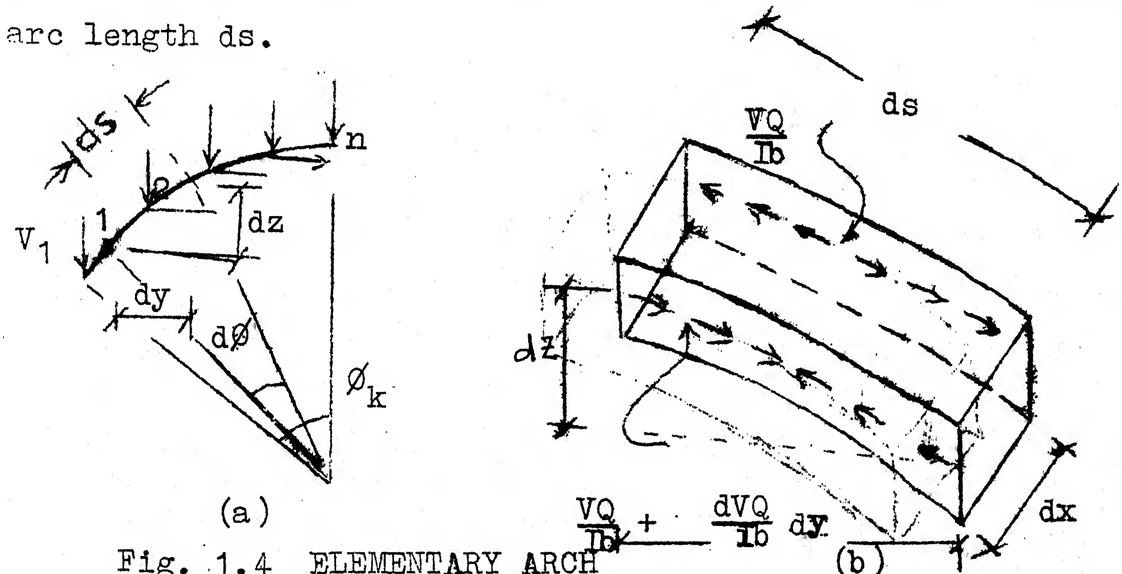


Fig. 1.4 ELEMENTARY ARCH STRIP

The arc length between two adjacent nodes i and i+1 is = ds (= $Rd\phi$.)

Vertical projection of ds = $dZ_i = R\ (\text{Cos}\ \phi_{i+1} - \text{Cos}\ \phi_i)$ (1.50)

Horizontal projection of ds= $dy_i = R(\text{Sin}\phi_i - \text{Sin}\phi_{i+1})$ (1.51)

Therefore, $$F_{vi} = q_u\ dy_i + q_g\ ds_i + \frac{\partial N_{x\phi}}{\partial x}\ dz_i \tag{1.52}$$

$$F_{hi} = \frac{\partial N_{x\phi}}{\partial x}\ dy_i \tag{1.53}$$

Therefore,

$$M_{\phi i} = \sum_{j=1,2}^{i} \left[F_{vj} (Y_i - Y_j) + F_{hj}(Z_i - Z_j) \right] \tag{1.54}$$

$$N_{\phi i} = (\sum_{j=1,2}^{i} F_{vj}) \sin \phi'_i - (\sum_{j=1,2}^{i} F_{hj}) \cos \phi'_i \tag{1.55}$$

$$Q_{\phi i} = (\sum_{j=1,2}^{i} F_{vj}) \cos \phi'_i - (\sum_{j=1,2}^{i} F_{hj}) \sin \phi_i \tag{1.56}$$

1.8 DISCUSSION;

The boundary conditions for arch calculations is determined from the physical system of the shell itself, which can be divided mainly in 3 types which are given below;

1) Single Barrel:- The arch may be considered as axisymmetric statically determinate system, loaded by external loads and supported by specific shear, having two ends free.

a) Type(1) b) Type(2) c) Type(3)

Fig.1.5 ARCH MODELS

2) Interior Shell:- The arch may be considered as axisymmetric statically inderminate system, loaded,as,type-1, with both ends fixed.

3) End Shell:- The arch may be considered as axi-asymetric statically determinate system, loaded as type 1, with one end fixed and another end free.

In the next chapter both the methods are illustrated by numerical examples.

CHAPTER-II

ANALYSIS OF CYLINDRICAL SHELL BY CLASSICAL AND BEAM THEORIES

2.1 INTRODUCTION:

The stress resultants and the displacements of a shell can be expressed through four constants a , b , c , d which can be determined from boundary conditions. For the shell without edge beam the boundary conditions are that N_x , $N_{x\phi}$, $N\phi$, M_ϕ are zero at the free edge. If the shell is provided with edge beam, the boundary conditions are the edge deformations of shell are same as those of edge beam at the interface of the shell and beam.

2.2 THEORY OF EDGE BEAM:

For the shell with edge beam the compatibility of deformations are:

1. Compatibility of vertical deformation
2. Compatibility of horizontal deformation
3. Rotational compatibility and
4. Longitudinal deformation compatibility.

If the stress resultants and the deformations are denoted by the subscript 'k' then,

$$\text{Vertical deformation, } w_{vs} = w_{rk} \sin \phi_k$$

$$\text{Horizontal deformation } w_{hs} = w_{rk} \cos \phi_k + u_{yk} \qquad (2.1)$$

$$\text{Rotational deformation } w_{ms} = \theta_k$$

$$\text{Longitudinal deformation } w_{xs} = u_{xk}$$

Loading of the edge beam at the interface is the edge stress resultants of the shell edge in same magnitude but of opposite sign along with the external loads, if any, and self weight of the edge beam. If V , H , M and S are the vertical, horizontal, rotational and shear loads respectively at the edge of the shell, the edge deformations are given by:

$$
\begin{aligned}
w_{vb} &= w_{vb} (V , H , M , S , q_b) \\
w_{hb} &= w_{hb} (V , H , M , S , q_b) \\
w_{mb} &= w_{mb} (V , H , M , S , q_b) \\
w_{xb} &= w_{xb} (V , H , M , S , q_b)
\end{aligned}
\qquad (2.2)
$$

Where q_b represents the external loading on the edge beam.

Again V , H , M , S , w_{vs} , w_{hs} , w_{ms} and w_{xs} are associated with a , b , c , d discussed in previous chapter and they can be expressed as functions of a , b , c , d if the loading and geometrical properties of shell and edge beam are known. Hence eqs. of 2.1 and 2.2 yield the interface compatibility as follows,

$$
\begin{aligned}
w_{vs} (a, b , c , d , q_s) &= w_{vb} (a , b , c , d , q_b) \\
w_{hs} (a , b , c , d , q_s) &= w_{hb} (a , b , c , d , q_b) \\
w_{ms} (a , b , c , d , q_s) &= w_{mb} (a , b , c , d , q_b) \\
w_{xs} (a , b , c, d, q_s) &= w_{xb} (a , b , c , d, q_b)
\end{aligned}
\qquad (2.3)
$$

where q_b and q_s are the total loading system on shell and edge beam. The four equations of 2.3 will now determine the four constants.

2.3 DEFORMATIONS OF EDGE BEAM:

Details of loading on edge beam are shown in Fig. 2.1. The expressions of deformations are given below;

a) Vertical Deformation:

$$w_{vV} = \frac{1}{m^2}\left[\frac{1}{m^2 E_b I_y} + \frac{(\pi - 2)e_h^2}{2 G_b J}\right] V = c_{b11} V$$

$$w_{vS} = -\frac{1}{m^3}\frac{1}{E_b I_y} e_v S = c_{b12} S$$

$$w_{vH} = -\frac{1}{m^2}\frac{\pi-2}{2G_b J} e_h e_v H = c_{b13} H \qquad (2.5)$$

$$w_{vM} = -\frac{1}{m^2}\frac{\pi-2}{2G_b J} e_h M = c_{b14} M$$

$$w_{vb} = w_{vV} + w_{vH} + w_{vM} + w_v$$

Where the first subscript indicates the direction of deflection and the second indicates the force causing it.

b) Longitudinal Deformation:

$$w_{xV} = -\frac{1}{m^2}\frac{1}{E_b I_y} e_v V = c_{b21} V$$

$$w_{xS} = -\frac{1}{m}\left[\frac{1}{A_b} + \frac{e_v^2}{E_b I_y} + \frac{e_h^2}{E_b I_z}\right] S = c_{b22} S$$

$$w_{xH} = -\frac{1}{m^3}\;\frac{1}{E_b\;I_z}\;e_{hH} \qquad = c_{b23}H \qquad (2.6)$$

$$w_{xM} = 0$$

$$w_{xb} = w_{xV} + w_{xS} + w_{xH}$$

c) Horizontal Deformation:

$$w_{hV} = -\frac{1}{m^2}\;\frac{\pi-2}{2G_bJ}\;e_h\;e_v\;V \qquad = c_{b31}V$$

$$w_{hS} = -\frac{1}{m^3}\;\frac{1}{E_b\;I_z}\;e_h\;S \qquad = c_{b32}S$$

$$w_{hH} = \frac{1}{m^2}\left[\frac{1}{m^2}\;\frac{1}{E_bI_z} + \frac{\pi-2}{2G_bJ}\;e_ve_h\right]H = c_{b33}H \qquad (2.7)$$

$$w_{hM} = -\frac{1}{m^2}\;\frac{\pi-2}{2G_b\;J}\;e_v\;M \qquad = c_{b34}M$$

$$w_{hb} = w_{hV} + w_{hS} + w_{hH} + w_{hM}$$

d) Rotational Deformation:

$$w_{mV} = -\frac{1}{m^2}\;\frac{1}{E_bI_y}\;e_v\;V \qquad = c_{b41}V$$

$$w_{mS} = 0$$

$$w_{mH} = -\frac{1}{m^2}\;\frac{\pi-2}{2G_b\;J}\,e_v\;H \qquad = c_{b43}H \qquad (2.8)$$

$$w_{mM} = \frac{1}{m^2}\;\frac{\pi-2}{2\;G_bJ}\;M \qquad = c_{b44}M$$

$$w_{mb} = w_{mV} + w_{mH} + w_{mM}$$

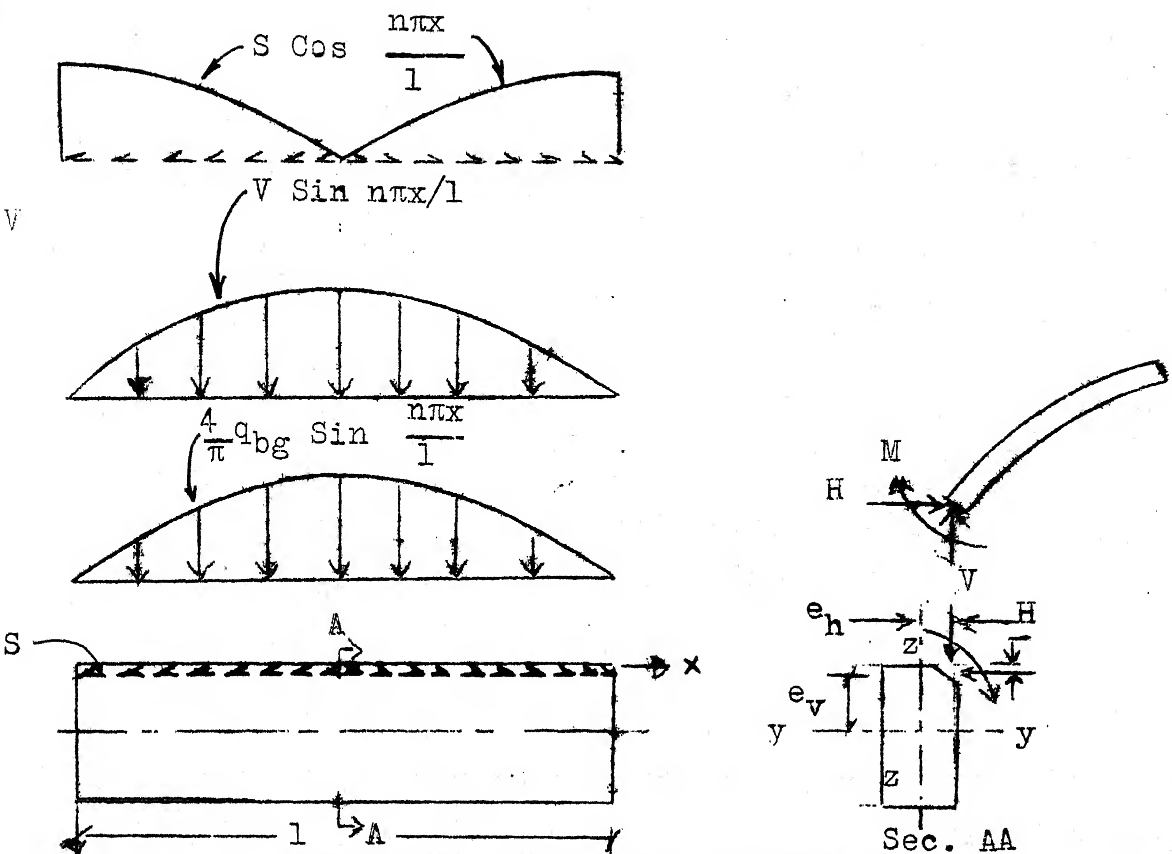


Fig. 2.1 LOADING ON EDGE BEAM

2.4 DEFORMATION OF EDGE BEAM DUE TO PRESTRESSING FORCE:

Prestressed cable in a vertical plane will cause deformation in vertical plane only. These deformations are vertical and longitudinal deformations. The edge beams are prestressed by curved cable (tendons) having parabolic profile. The effect of prestressing the edge beams is equivalent to the particular cases of loading which are described below and shown in Fig. 2.2.

1. Continuous distributed upward pressure(w_{be}) due to the change ~~of~~ curvature of the cable profile and if magnitude is given by,

$$q_{be} = \frac{4}{\pi} \quad \frac{8\ g\ p_e}{l^2} \qquad (2.9)$$

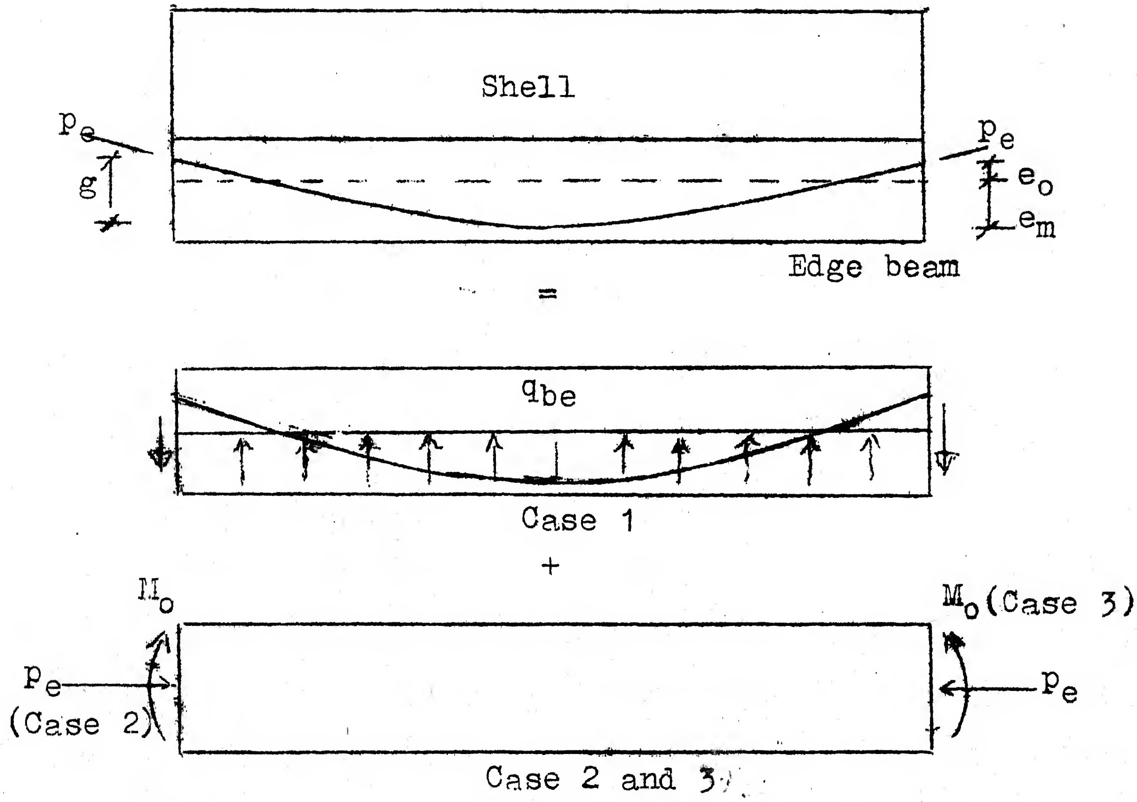


Fig. 2.2 EFFECT OF PRESTRESSING FORCE

2. End moment due to eccentricity of anchorage is given by,

$$M_o = \frac{4}{\pi} \quad p_e \quad e_o \qquad (2.10)$$

3. Axial thrust applied at both ends of the centre of gravity of the beam section.given by, axial thrust = $\frac{4}{\pi} \quad \frac{p_e}{A_b}$

(2.11)

Vertical deformation caused by prestressing force,

$$w_{vp} = - \frac{4}{\pi}\left(\frac{1}{m^4} \frac{8g}{E_b I_y} - \frac{l^2}{m^2} \frac{e_o}{E_b I_y} \right) \frac{p_e}{l^2} \qquad (2.12)$$

and longitudinal deformation,

$$w_{xp} = - \frac{4}{\pi}\left(\frac{1}{A_b} + \frac{e_o e_v}{E_b I_y} - \frac{1}{m^2} \frac{8g\ e_v}{l^2} \frac{1}{E_b\ I_y} \right) p_e \qquad (2.13)$$

The coefficient $4/\pi$ is used in all the Eqs. 2.12 and 2.13 as a Fourier coefficient of uniformly distributed load converted into sine series.

2.5 ANALYSIS OF SHELL BY CLASSICAL THEORY:

The theory described in article 1.6 is the theoretical aspect of cylindrical shell analysis by the method described in ASCE Manual-31. A series of comprehensive tables are given in ASCE Manual-31 for barrel roof analysis. In this investigation shells are analysed using those tables taking n=1 .

All the coefficients given in the table of ASCE Manual-31 for the loads, assuming sinusoidal variation of loads along the longitudinal direction. The method to analyse the shell is given below,

Membrane Analysis:-

$$N_x = \frac{4}{\pi} \left(\frac{1}{R}\right)^2 q\ R c_{1m} \operatorname{Sin} mx \qquad (2.14)$$

$$N_{x\phi} = \frac{4}{\pi} \left(\frac{1}{R}\right) q\ R\ c_{2m} \operatorname{Cos} mx$$

$$N_{\phi} = \frac{4}{\pi} q R c_{3m} \sin mx$$

Where q is the loading. For $q = q_g$, c_{1m} , c_{2m} , c_{3m} are given in column 7 , 8 , 9 of Table 1 B and for $q = q_u$ column 1 , 2 , 3 gives the above coefficients. If $c_s = l^4/(R^3 t E_s)$, then the membrane displacement can be obtained from the following expression,

i) For q_g

$$w_{vm} = q_g R c_s \left[\left(\frac{2R}{\pi l} \right)^2 + \frac{2}{\pi^4} + \left(\frac{R}{l} \right)^4 c_{w1m} \right] \sin mx \qquad (2.15)$$

$$w_{hm} = q_g R c_s \left[\left(\frac{R}{l} \right)^4 c_{w2m} \sin m x \right]$$

c_{w1m} and c_{w2m} are given in column 10 and 11 of Table 1 B.

ii) For $q = q_u$

$$w_{vm} = q_u R c_s \left[\left\{ 1 + \frac{1}{2} (\pi R/l)^2 + (\pi R/l)^4/12 \right\} c_{w1m} \right] \sin mx$$

$$w_{hm} = q_u R c_s \left[(R/l)^4 c_{w2m1} + \left\{ 1 + (\pi R/l)^2/2 + (\pi R/l)^4/12 \right\} \times c_{w2m2} \right] \sin mx \qquad (2.16)$$

c_{w1m}, c_{w2m1} and c_{w2m2} are given in column 4 , 5 and 6 of Table 1 B and the longitudinal deformation,

$$w_{xm} = \frac{1}{E_s t} (N_x) / (l/R)^2 = \frac{1}{E_s t} f_{osm} \qquad (2.17)$$

Errors and Correction:- To satisfy the boundary condition i.e. N_x , $N_{x\phi}$, N_{ϕ} at shell edge are zero, a set of line loads

should be applied at the edge which is equal in magnitude but opposite in sign of N_{xk} , $N_{x\phi k}$ and $N_{\phi k}$. Correction forces are resolved into vertical and horizontal components and they are:

$$V_L = - N_{\phi k} \text{ Sin } \phi \text{ Sin } m x$$
$$H_L = - N_{\phi k} \text{ Cos } \phi \text{ Sin } m x \qquad (2.18)$$
$$S_L = - N_{x\phi k} \text{ Cos } m x$$

Bending Analysis:-

The correction line loads will disturb the membrane state of shell and bending of shell will occur. The bending analysis of shell is done in following way.

Edge deformations due to correction line loads

$$w_{vL} = c_s \left[c_{v1} V_L + c_{v2} S_L + c_{v3} H_L \right] \text{Sin } mx$$
$$w_{xL} = \left(\frac{1}{R}\right)^2 \frac{1}{tE_s} \left[c_{11} V_L + c_{12} S_L + c_{13} H_L \right] \text{Sin } mx \qquad (2.19)$$
$$w_{ml} = \frac{R^2}{E_s I} \left[c_{m1} V_L + c_{m2} S_L + c_{m3} H_L \right] \text{ Sin } mx$$
$$w_{hL} = c_s \quad c_{h1} V_L + c_{h2} S_L + c_{h3} H_L \quad \text{Sin } mx$$

Where c_{v1}, c_{v3}, c_{v2}, c_{hi}, c_{h3} , c_{m1} , c_{m3} and c_{m2} are given in column 1, 4, 7; 2, 5, 8; 3, 6 and 9 of Table 2B and c_{11}, c_{13} c_{12} are given in column 1 , 5 and 9 of Table 2A.

When the shell is provided with edge beam a set of forces, say V_b, H_b, S_b and M_b, will generate at the interface. The deformations due to the interface forces are:

$$w_{vbl} = c_s \left[c_{v1} V_b + c_{v2} S_b + c_{v3} H_b + c_{v4} M_b/R \right] \text{Sin } mx$$

$$w_{xbl} = (1/R)^2 \, 1/E_s t \left[c_{11} V_b + c_{12} S_b + c_{13} H_b + c_{14} M_b/R \right] \text{Sin } mx$$

$$w_{hbl} = c_s \left[c_{h1} V_b + c_{h2} S_b + c_{h3} H_b + c_{h4} M_b/R \right] \text{Sin } mx \qquad (2.20)$$

$$w_{mbl} = \frac{R^2}{E_s I} \left[c_{m1} V_b + c_{m2} S_b + c_{m3} H_b + c_{m4} M_b/R \right] \text{Sin } mx$$

where c_{m1}, c_{m3}, c_{m2}, c_{v4}, c_{h4} and c_{m4} are given in column 3, 6, 9, 10, 11 and 12 of Table 2 B and c_{14} in column 4 in Table 2 A. If positive interface load acts on the shell then negetive load will act on the beam, then the deformation compatibility at mid span of shell edge yields,

$$\begin{aligned}
w_{vm} + w_{vL} + w_{vbl} - w_{vb} &= 0 \\
w_{xm} + w_{xL} + w_{xbl} - w_{xb} &= 0 \\
w_{hm} + w_{hL} + w_{hbl} - w_{hb} &= 0 \\
w_{mL} + w_{mbl} - w_{mb} &= 0
\end{aligned} \qquad (2.21)$$

(3rd Subscript 1 indicates deformations due to loading)

$$\begin{bmatrix}
c_s c_{v1} + c_{b11} & c_s c_{v2} + c_{b12} & c_s c_{v3} + c_{b13} & \frac{c_s c_{v4}}{R} c_{b14} \\
(1/R)^2 \frac{1}{E_s t} c_{11} + c_{b21} & (\frac{1}{R})^2 \frac{1}{E_s t} c_{12} + c_{b22} & (\frac{1}{R})^2 \frac{1}{E_s t} c_{13} + c_{b23} & (\frac{1}{R})^2 \frac{c_{14}}{E_s t} \\
c_s c_{h1} + c_{b31} & c_s c_{h2} + c_{b32} & c_s c_{h3} + c_{b33} & \frac{c_s c_{h4}}{R} + c_{b34} \\
\frac{R^2}{E_s I} c_{m1} + c_{b41} & \frac{R^2}{E_s I} c_{m2} & \frac{R^2}{E_s I} c_{m3} + c_{b43} & \frac{c_s c_{m4}}{R} + c_{b44}
\end{bmatrix} \times \begin{bmatrix} V_b \\ S_b \\ H_b \\ M_b \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}$$

(2.22)

Where,

$$C_1 = w_{vm} - w_{v\ell} + w_{vbg} + w_{vbp}$$

$$C_2 = -w_{xm} - w_{x\ell} + w_{xbg} + w_{xbp}$$

$$C_3 = -w_{hm} - w_{h\ell} \text{ and } C_4 = -w_{m\ell} \qquad (2.23)$$

w_{vbg} = vertical deformation due to self weight of the beam

$$= c_{b11}\, b\, d\, \gamma \qquad (2.24)$$

w_{xbg} = longitudinal deformation due to the self weight of the beam = $c_{b21}\, b\, d\, \gamma$ (2.25)

Analysis of shell by using the formulae direct from the solution of differential equation of shell involves the evaluation of the constants a , b , c and d as basic unknowns whereas method using the tables takes the interface forces as basic knowns, which are determined from the boundery conditions. Once V_b , S_b , H_b , and M_b are known, the total correction loads;

$$V = V_L + S_b \quad , \quad S = S_L + S_b \ , \ H = H_L + H_b , \ M = M_L + M_b \qquad (2.26)$$

Stress resultants due to bending;

$$N_x = \left(c_{11} V + c_{12} S + c_{13} H + c_{14} \frac{M}{R} \right) \times (1/R)^2$$

$$N_{x\phi} = \left(c_{21} V + c_{22} S + c_{23} H + c_{24} \frac{M}{R} \right) \times (1/R)$$

$$N_\phi = c_{31} V + c_{32} S + c_{33} H + c_{34} M/R \qquad (2.27)$$

$$M_\phi = (c_{41} V + c_{42} S + c_{43} H + c_{44} M/R)R$$

Total stress resultants can be obtained by super-imposing membrane stress resultants obtained from Eqs. of 2.14 to bending stress resultant obtained from Eqs. of 2.27.

It is general practice for single barrel shell to assume that edge of the shell does not offer any restraint against horizontal and rotational deformations, so Eqns. of 2.22 reduces to 2 simultaneous equations. For the interior shell of a multi barrel group $w_h = 0$ and $w_m = 0$ while for single shell $H = 0$; $M = 0$. Numerical computations of the method is illustrated by the following example.

2.6 EXAMPLE 1

$l = 40$ m , $R = 12$ m , $t = 0.12$m , $b = 0.15$ m , $d = 2.0$ m.

$\phi_k = 40^0$, $e_o = 0.5$ m , $c_m = 0.7$ m , $p_e = 400,000$ kg.

Loading:

i) Live load on shell = 75 kg/m^2 (acting on surface of shell)

ii) Load from lime terracing = 45 kg/m^2 (acting on surface of shell)

iii) Self weight of shell = 2400 x 0.12 = 360 kg/m^2

Total load on the shell = 480 kg/m^2

Grade of concrete of shell and beam is M_{350}.

The shell is single barrel

a) Membrane Analysis:-

Membrane stresses are calculated from Eqs. of 2.14 and are given in Table 2.1.

Table 2.1 MEMBRANE STRESS RESULTANTS

ϕ	N_x (kg/m)	$N_{x\phi}$ (kg/m)	N_ϕ (kg/m)	M_ϕ (kg/m)
40	-14040	0	-6234	0
30	-13820	-2297	-6139	0
20	-13190	-4524	-5858	0
10	-12160	-6614	-5399	0
0	-10750	-8503	-4775	0

b) Errors and Corrections:

Errors are $N_{\phi k}$ = - 4775 kg/m and $N_{x\phi k}$ = - 8503 kg/m . The vertical and horizontal component shear correction line loads are calculated from Eqs. of 2.14.

V_L = 3070 kg/m

H_L = 3658 kg/m

S_L = 8503 kg/m

c) Bending Analysis:

i) Deformation of shell edge at mid span:- It is general practice to ignore the restraint offered by the edge

beam towards the horizontal and rotational deformation as the beam is slender one. So for this problem H_b and M_b may be taken as zero. From Eqs. 2.15 , 2.17 and 2.19.

$$w_{vm} + w_{v1} = c_s \; 54500$$

$$w_{xm} + w_{x1} = \left(\frac{l}{R}\right)^2 \frac{1}{E_s t} \; 26130$$

ii) Deformation of Edge Beam:- The contribution of e_h towards the beam deformation is very small and hence ignored.

From Eqs. of 2.5

$$w_{vvb} + w_{vbs} = c_{b1} \; (- 0.12319) \; V_b + \frac{h}{l} \; 0.19351 \; S_b$$

From Eqs. of 2.25

$$w_{vbg} = c_{b1} \; 112.9$$

From Eq. 2.12

$$w_{vbp} = c_{b1} \; 1.2548 \; e_m - 0.2923 \; e_o$$

From Eqn of 2.6

$$w_{xvb} + w_{xbs} = c_{b2} \; (0.60793) \; V_b - \frac{d}{L} \; 1.2732 \; S_b$$

$$w_{xbg} = c_{b2} \; 557.3$$

From Eq. 2.13

$$w_{xbp} = - c_{b2} \; (1.2732 \; d + 1.4478 \; e_o - 6.1923 \; e_m) \; pe/l^2$$

Where , $c_{b1} = \dfrac{l^4}{bE_b d^3}$ and $c_{b2} = \dfrac{l^2}{bE_b d^2}$

iii) Compatibility of Deformations:- Vertical deformation compatibility from Eq. 2.21 ,

$$42.38\ V_b + 0.9561\ S_b - \frac{t}{b}\left(\frac{R}{d}\right)^3 \left(-0.1231\ V_b + \frac{d}{l}\ 0.19351\ S_b\right.$$

$$\left. + 112.9 + (1.2548\ e_m - 0.2923\ e_o)\ p_e/L^2\right) + 54500 = 0 \qquad (2.28)$$

Longitudinal compatibility from Eq. 2.21

$$-0.216\ V_b - 0.1668\ S_b - \frac{t}{b}\left(\frac{R}{d}\right)^2 \left(0.60793\ V_b - \frac{d}{l} 0.12732\ S_b\right.$$

$$\left. - 557.3 + (1.2732\ d_t\ 1.4478\ e_o - 6.1923\ e_m)\ \frac{p_e}{L^2}\right) + 26130 = 0 \qquad (2.29)$$

d) <u>Total Stress Resultants</u>:

Eqs. 2.28 and 2.29 gives $V_b = -1226$ and $S_b = -15980$

Therefore, final correction line loads from eqs. of 2.26,

$V = 1844$, $S = -7477$ and $H = 3658$

Bending stress resultants are calculated from Eqs. of 2.27 and adding these with membrane solution total stress resultants are obtained and given in Table 2.2.

Table 2.2 TOTAL STRESS RESULTANTS

ϕ'	N_x (kg/m) (at x=l/2)	$N_{x\phi}$ (kg/m) (at x = 0)	N_ϕ (kg/m) (at x= l/2)	M_ϕ (kgm/m) (at x=l/2)
40	-51180	0	-5393	735
30	-43180	-10100	-4919	695
20	-22950	-17090	-3674	991
10	-27290	-19610	-2100	865
0	-44570	-19720	-619	0

Longitudinal stress at the bottom of the beam

$$\frac{N_{xb}}{b} = \frac{l^2}{b\ d^2}\left[0.60793\ \left(\frac{4}{\pi} b\ d\ \sqrt{\ } - V_b\right) + 0.6336\ \frac{d}{l}\ S_b + \frac{4}{\pi}\left(-d + 1.4478\ e_o - 6.1923\ e_m\right)\right] \qquad (2.30)$$

$$= -\ 297000\ (\ kg/m^2\ )$$

The variation of stress resultants are shown in Fig. 2.5 (a) and (b).

2.7 ANALYSIS OF SHELL BY BEAM THEORY:

Theoretical aspect of beam theory has been described in the article1.7.The beam method may be devided into two parts as beam and arch analysis. To illustrate the numerical computa-tations of the method the same shell of example 1 is analysed below.

a) Beam Analysis:

The transverse section of the shell along with the edge beam is assumed to constiute an uncracked section of a beam whose geometrical properties are shown in Fig. 2.3 .

Chord length, $l_c = 2R \sin \phi_k = 15.4286$ m

Depth of the section, $h = R\ (\ 1- \cos \phi_k\ ) + d = 4.8075$ m

Area of the Section, $A_s = 2\ (\ A_C + A_B\ ) = 2.6106\ m^2$

Distance of the C.G. from crown, $Z_c = \dfrac{2(A_c\ Z_c + A_b\ Z_b\ )}{A_s} = 1.6077$ m

Moment of inertia of the section, $I_s = 2\ (\ I_b + I_c + A_b\ Z_{rb}^2 +$

$A_c\ Z_{rc}^2\ ) = 5.3950\ m^4$

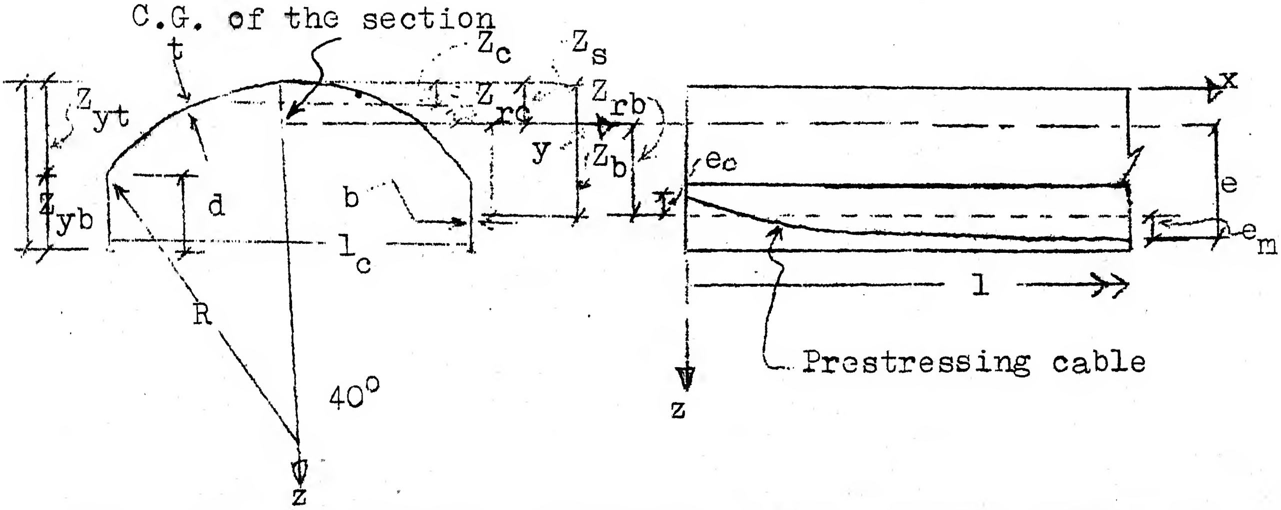


Fig. 2.3 GEOMETRICAL PROPERTIES OF ASSUMED BEAM SECTION OF EXAMPLE 1

First moment of area of any section, ϕ'

i) for shell portion, $Q = 2Rt\ \ Z_s - R\ (1 - \sin \phi_k/\phi_k\ )$

ii) for beam portion, $Q = \int_{Z}^{Z_{yb}} 2b\ Zdz = b(Z_{yb}^2 - Z^2\ )$

Co-ordinates of any point on the shell,

$Z_i = R\ (\ 1 - \cos \phi_i\ ) - Z_s$

$Y_i = R \sin \phi_i$

Total load U.D.L. on the shell on the chord area,

$q_o = \left[\ (A_s\ \gamma\ + q_g\ )\ \phi_k/\sin \phi_k + q_u \right]\ l_c = 8277.4946$ kg/m

Bending moment at the mid span,

$$M = \frac{q_o l^2}{8} = 1655.2 \times 10^3 \text{ kg. m.}$$

Shear force at the end section,

$$S_f = \frac{q_t L}{2} \; 165.52 \times 10^3 \text{ kg.}$$

N_x/t at the top of the section

$$= \frac{p_e}{A_s} - \frac{p_e\,e}{I_s} Z_{yt} + \frac{M}{I_s} Z_{yt} = 107700$$

N_x/b at the bottom of the section,

$$= -\frac{p_e}{A_s} - \frac{p_e\,e}{I_s} Z_{yb} + \frac{M}{I_s} Z_{yb} = -702635$$

$N_{x\phi}$ at any section ϕ' $= \dfrac{S_f\,Q}{2\,I_s}$

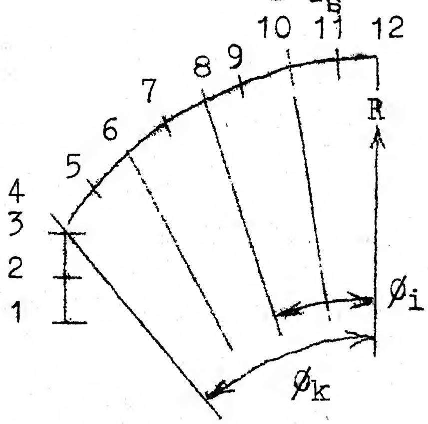


Fig. 2.4 NODES ASSUMPTIONS FOR ARCH ANALYSIS

The section is a symmetric section, so the stresses of the half section is given in the Table 2.3

Table 2.3 σ_x AND $N_{x\phi}$ OF EX.2 (BY BEAM METHOD)

Nodal Points	ϕ°	Co-ordinates Z_i	Y_i	Q	σ_x (kg/m^2) (at x= L/2)	$N_{x\phi}$ (kg/m) (at x=L/2)
1	-	3.2028	7.7136	0.000	107700	0000
2	-	2.2028	7.7136	0.8099		5218
3	-	1.2028	7.7136	1.320		8504
4	40	1.2028	7.7136	1.320		8504
5	35	0.5644	6.8832	1.540		9921
6	30	0.0028	6.0	1.6093		10370
7	25	-0.4804	5.0712	1.5461		9963
8	20	-0.8816	4.1040	1.3732		8845
9	15	-1.196	3.1056	1.1089		7146
10	10	-1.4228	2.0832	0.7775		5010
11	5	-1.5596	1.0464	0.4002		2570
12	0	-1.6077	0.0000	0.0000	702635	0000

b) Arch Analysis:

An elementry arch strip of width dx is considered at midspan, which containts the shell and the edge beam together and divided into 12 nodal points as shown in Figure 2. The details of arch calculations are given below in tabular form Table 2.4 contains ds , dy , dz , $\frac{\partial N_{x\phi}}{\partial x}$. Table 2.5

contains $\frac{\partial N_{x\phi}}{\partial x}$ dz , q_g ds , q_u dy , F_v , F_h (= $\frac{\partial N_{x\phi}}{\partial x}$ dy) and Table 2.6 contains N_ϕ , M_ϕ , distance of the nodes from the top. It is assumed that nodes 4 and 12 have half of the segmental length and 1 & 3 have segmental length $\frac{1}{4}$ d where 2 has d/2.

Table 2.4 ARCH ANALYSIS

Nodes	$\frac{\partial N_{x\phi}}{\partial x}$	ds	dy	dz
1	000.0000	0.5000	0.00000	0.50000
2	260.8898	1.0000	0.00000	1.00000
3	425.7712	0.5000	0.00000	0.50000
4	424.7712	0.5236	0.88280	0.39528
5	494.9012	1.0472	0.85752	0.60048
6	517.4841	1.0472	0.90660	0.52344
7	526.5362	1.0472	0.94884	0.44244
8	441.0083	1.0472	0.98364	0.35808
9	356.7043	1.0472	1.01120	0.27096
10	250.0121	1.0472	1.03920	0.18168
11	128.9848	1.0472	1.04280	0.09132
12	000.0000	0.5236	0.52334	0.01140

Table 2.5 ARCH ANALYSIS

Nodes	$w_g^* d_s$ (1)	$w_u d_y$ (2)	$\frac{\partial N_{x\phi}}{\partial x} dz$ (3)	F_y (1)+(2)+(3)	F_h $(\frac{\partial N_{x\phi}}{\partial x} dy)$
1	213.6288	0	000.0000	213.6288	000.0000
2	427.2576	0	- 11.7789	415.4787	000.0000
3	427.2576	0	- 45.4222	381.8325	000.0000
4	427.2576	0	- 96.6527	330.6049	207.0933
5	427.2576	0	-157.9163	269.3413	424.3885
6	427.2576	0	-232.9607	194.2969	469.1511
7	427.2576	0	-270.8705	156.3871	499.5986
8	427.2576	0	-297.1789	130.0787	434.7934
9	213.6288	0	-167.64932	74.7334	360.6997
10	180.0000	0	-212.8856	-32.8856	257.7425
11	360.0000	0	-260.8898	99.1102	134.5054
12	180.0000	0	000.0000		000.0000

w_g for shell 408 kg/m^2 and for beam 360 kg/m^2

$w_g = q_g$

$w_u = q_u$

Table 2.6 N_ϕ AND M_ϕ (BY BEAM METHOD)

Nodes	h_i	N_ϕ kg/m (at x = L/2)	M_ϕ kg m/m (at x= L/2)
1	4.8110	0000	0000.0000
2	3.8110	0000	0000.0000
3	2.8110	0000	0000.0000
4	2.8110	-1430	0000.0000
5	2.1726	-1580	1635.1483
6	1.5954	-1804	2961.8528
7	1.1248	-2136	4094.0868
8	0.7236	-2387	4997.2154
9	0.3560	-2591	5683.6779
10	0.1754	-2712	6160.6666
11	0.0456	-2794	6428.3211
12	0.0000	-2887	6508.4988

2.8 DISCUSSION:

It is evident from the articles 2.6 and 2.7 that there is no agreement between the stress resultants evaluated by beam method and classical method. A comparative study for the structural behaviour determined by both the theories is necessary to get the idea about the error involvement in beam theory. The influence of the different shell parameters,

prestressing force and eccentricities of the cable should be investigated. In next chapter(3) a comparative study of the structural behaviour of the shell of example 2 and parametric study are done.

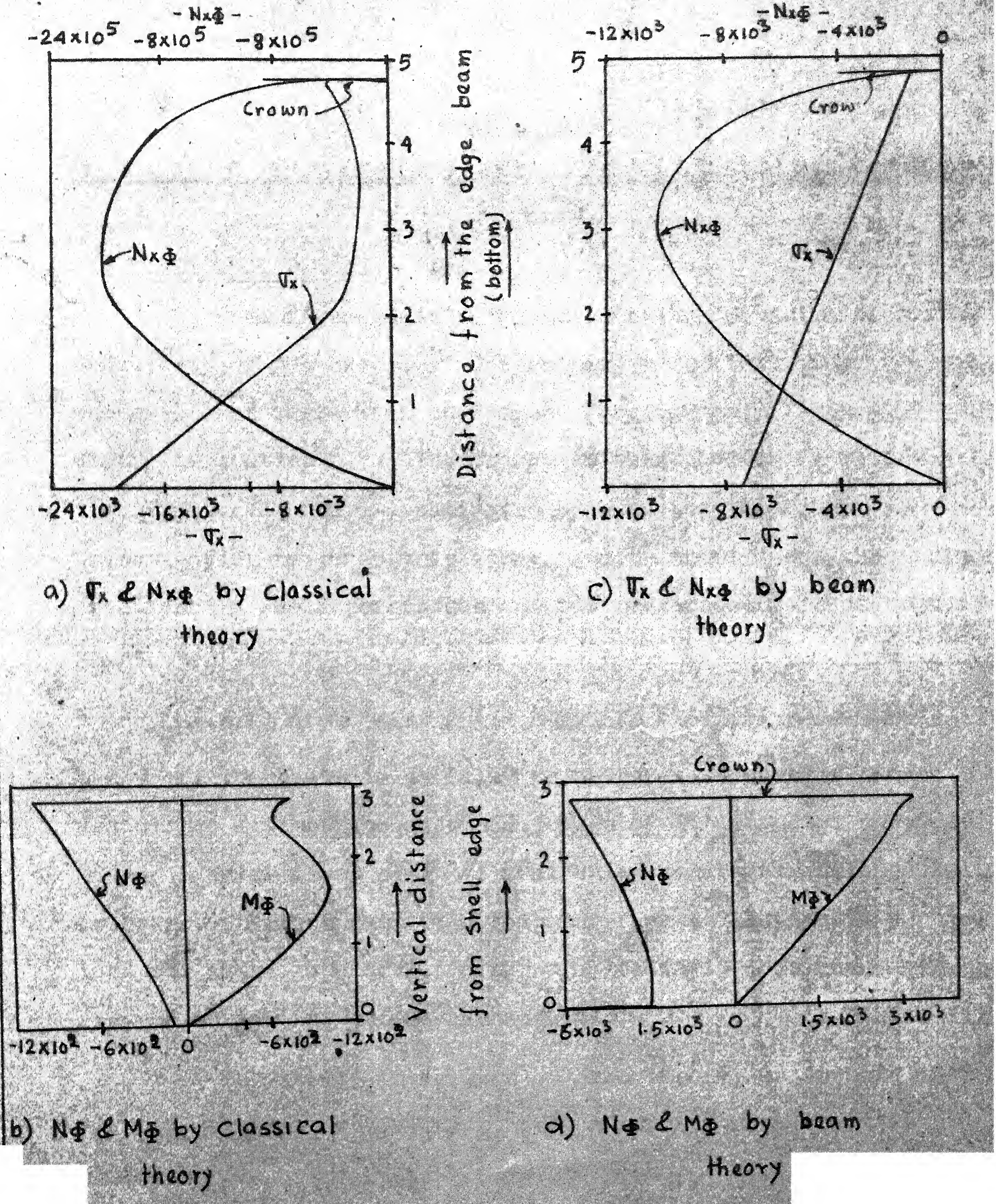


Fig. 2.5 VARIATION OF STRESS RESULTANTS (Ex.-1)

CHAPTER-III

COMPARATIVE STUDY OF STRUCTURAL BEHAVIOUR BY BEAM AND CLASSICAL THEORIES

3.1 INTRODUCTION:

From figure2.25 it is found that the variation of stress resultants of the example-1 determined in section 2.6 by classical theory do not agree with the stress resultants determined by beam theory in section 2.7. The object of this chapter is to investiga the structural behaviour estimated by both the theories. Investigation by parametric study is done to find out the influenc of different shell parameters and the prestressing force on the structrual behavior.

3.2 ANALYSIS OF SHELL IN FULLY MEMBRANE STATE BY BEAM THEORY(2):

The shell is in fully membrane state when it offers resistance against the external forces by it's inplane stresses only. A cylindrical shell with circular generatrix attains fully membrane condition when it's transverse cross section is continuo i.e. the shape is circular ring and loading is continuous with respect to its transverse cross section.

The shell, to analyse by beam theory, is assumed as a beam spanning in the direction of it's axis of rotation with a hollow circular transverse cross section.

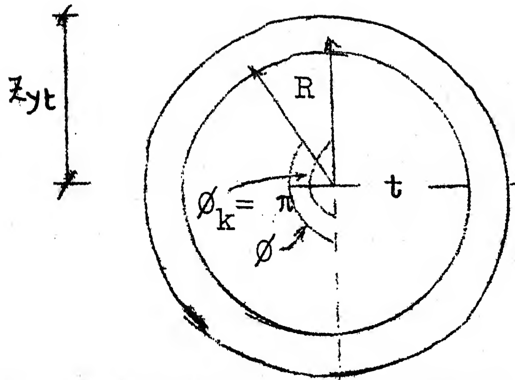


Fig. 3.1 CIRCULAR RING CROSS SECTION

Loading on the shell $= 2\pi Rq_g$ Sin mx

Moment of Inertia, $I_c = \pi R^3 t$

First moment of area at any section ϕ, $= 2R^2 t \sin \phi$

Bending moment at any lognitudinal section, $M = 2\pi R\, q_g \cdot \frac{1}{m^2}$ Sin mx

Shear force $S_f = -\frac{1}{m} 2\pi R\, q_g \cos mx$

$Z_{yt} = R \cos(\pi/2 - \phi) = -R \cos \phi$

Longitudinal stress, $\sigma_x = -2\pi Rq_g \frac{1}{m^2} \sin mx \frac{-R\cos\phi}{\pi R^3 t}$

Shear stress, $\tau = -2\pi Rq_g \frac{1}{m} \cos mx \frac{2R^2 t \sin\phi}{2\pi t R^3 t}$ (3.1

Trans verse force, $N_\phi = -q_g \sin mx\, (-R\cos\phi_k)$

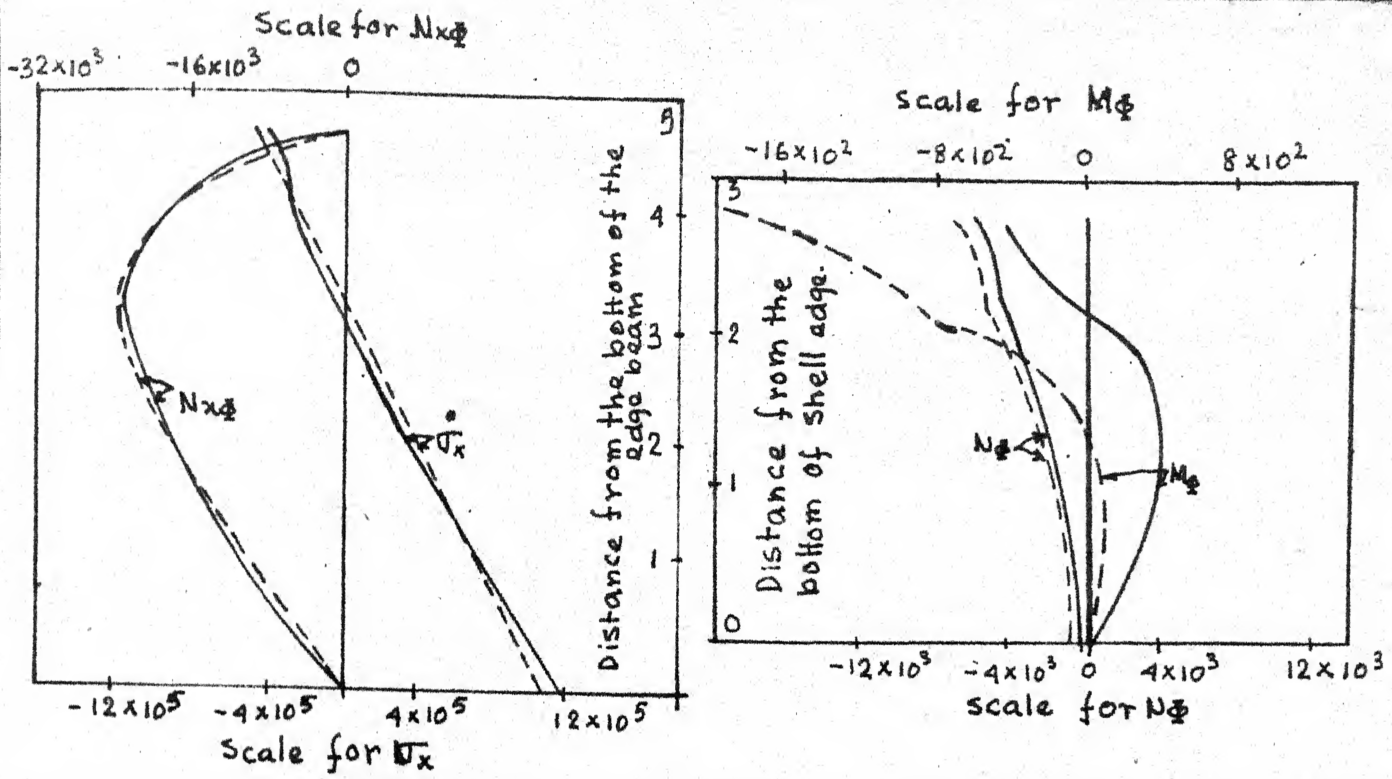
Substituting $N_x = \sigma_x t$, and $N_{x\phi} = \tau\, t$ and $m_x = n\pi/l$ in eqn. of 3.1.

$$N_x = q_g R \left(\frac{l}{R}\right)^2 \cos\phi \cdot \frac{2}{n^2\pi^2} \sin \frac{n\pi x}{l}$$

$$N_{x\phi} = -q_g R \left(\frac{l}{R}\right) \sin\phi \frac{2}{n\pi} \cos \frac{n\pi x}{l} \qquad (3.2)$$

$$N_\phi = q_g R \cos\phi \sin \frac{n\pi x}{l}$$

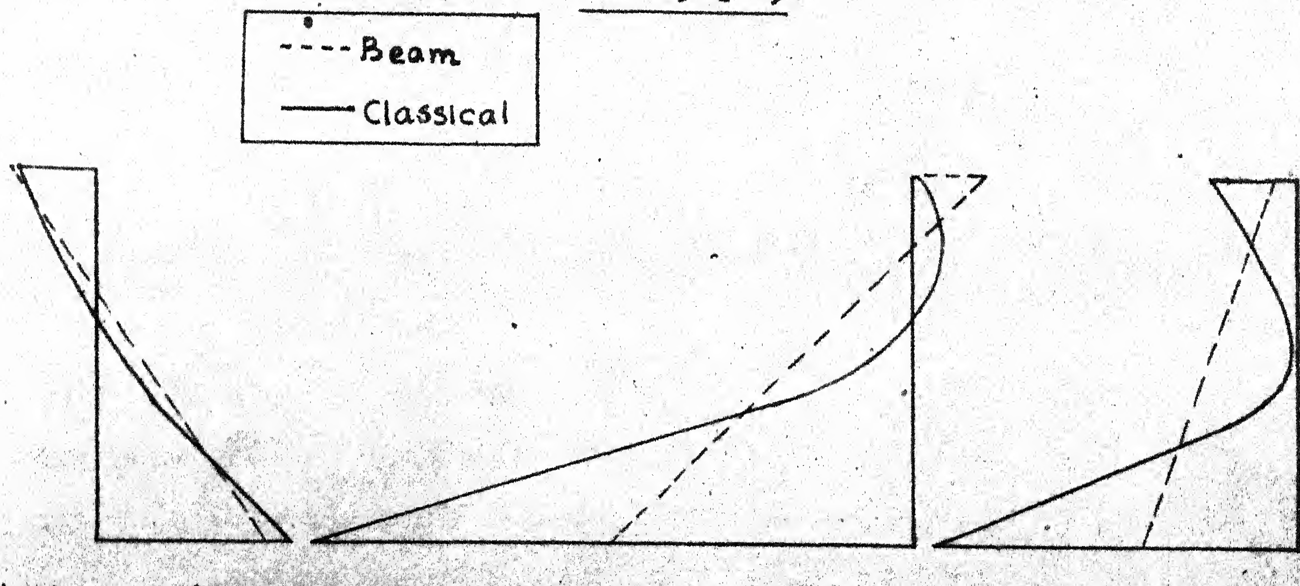
If $\phi_k = \pi$ is substituted then eqns. 1.7, 1.8, 1.9 would give exact same expressions like eqn. of 3.2



a) σ_x and $N_{x\Phi}$ b) N_Φ and M_Φ

Fig. 3.2. VARIATION OF STRESS RESULTANTS

(Ex.1, $P_e=0$)



Contribution of q_u & q_q Contribution of P_e Total

Fig. 3.3 CONTRIBUTION OF q_u q_q AND P_e TOWARDS σ_x

3.3 COMPARATIVE STUDY OF THE STRUCTURAL BEHAVIOUR

By the term structural behaviour it is meant the response of the shell by stress resultants and the deformations. As the investigation mainly aims to explore the feasibility of the aplicability of beam theory to design cylindrical shell the following parameters are considered for structural behaviour:

σ_x, N_ϕ, M_ϕ at mid span and $N_{x\phi}$ at support.

a) Stress Resultants

i) σ_x :- The variation of σ_x obtained by classical method is not linear while beam method gives linear distribution. For both the cases magnitude at the bottom of edge beam is greater than that of crown and the ratio of σ_x at crown to that at bottom of edge beam for classical method is 0.1765 and for beam method is 0.1222. Again σ_x at crown by exact method is 3.287 times greater than that by beam method and at the bottom of edge beam the ratio is 2.805. For the case of classical method magnitude decreases upto the section in between 20°to 10° then bifurcate towards the increasing mode and attains it's maximum value at the bottom of edge beam but for the beam method it follows through out the same path to attains the maximumat the bottom of edge beam.

ii) $N_{x\phi}$:- The mode of the variation of $N_{x\phi}$ along the depth of the edge beam is approximately similar with a difference that the rate of change of $N_{x\phi}$ at the region where it attains it's maximum

of magnitude for the classical theory is smaller than that for the beam theory but the main difference is that the magnitude obtained by the beam method is 50 to 60% less than that obtained by classical method.

iii) N_ϕ : The mode of the variation of N_ϕ exhibits it's tendency to decrease towards the edge of shell both for classical and beam method. At the crown N_ϕ obtained by the beam theory is 0.1945 times lesser than that obtained by classical theory while at the shell edge it is 2.318 times greater.

iv) M_ϕ : Sign of M_ϕ, obtained by both the theories is positive. The mode of variation shows the decrease in magnitude as it proceeds from crown and ultimately 0 at the shell edge. For classical theory magnitude decreases upto a section where ϕ is 30° and then increases upto section $\phi = 40°$. From this point the magnitude again decreases and comes to 0 at the bottom of the edge beam. Beam theory always gives the lower value of M_ϕ than classical theory.

b) Deformations

The defromations may be calculated by the classical theory from the eqns. of 2.19 by putting V, S, H, M in place of V_2, S_2, H_2 and M_2, where eqn. of 2.26 gives the values of V, S, H and M. The deformations of the example 2 is calculated as follows,

$$w_v = c_s \quad c_{v1}V + c_{v2}S + c_{v3}H$$

$$= \frac{l^4}{R^3 \ tE_s}\left[42.38 \times 1744 - 22.99 \times 3568 + 0.9856 \ (-7477)\right]$$

$$= -48 \text{ m.m.}$$

$$w_h = c_s \quad c_{h1}V + c_{h2}S + c_{h3}H$$

$$= \frac{l^4}{R^3 tE_s}\left[22.99 \times 1744 - 13.59 \times 3658 + 0.2931 \ (7477)\right]$$

$$= -35 \text{ m.m}$$

$$w_m = \frac{R^2}{E_s I} \qquad c_{m1}V + c_{m2}S + c_{m3}H$$

$$= \frac{R^2}{E_s I}\left[0.1245 \times 1744 - 0.07595 \times 3568 + 0.001413 \ (-7477)\right]$$

$$= -0.017 \text{ radians.}$$

Deformations by beam theory are calculated as follow:

$$w_v = \frac{4}{\pi} \ \frac{l^4}{E_s I_s}\left[\frac{l^4}{\pi^4}\left(q_t - \frac{8g \ P_e}{l^2}\right) - \frac{l^2}{\pi^2} \ P_e \ e_e\right]$$

$$= 3.5 \text{ m m}$$

w_h and w_m can not be determined by beam theory as it neglect those quantities by virtue of first assumptions of article 1.7(a).

Beam theory gives the magnitude of w_v less than that of classical theory, as in this case w_v obtained by classical theory is 13.21 times greater than that determined by beam theory in opposite direction. Beam theory gives w_h and w_m negligible

where as for classical theory they are not negligible: actually

$\delta_{vh} = 0.728$ and $\delta_{vm} = 1.9$

where,

$$\delta_{vh} = \frac{(\bar{w}_h/L) \text{ obtained by classical theory}}{(\bar{w}_v/L) \text{ obtained by classical theory}}$$

$$\delta_{vm} = \frac{(\bar{w}_m/\phi_k) \text{ obtained by classical theory}}{(\bar{w}_v/L) \text{ obtained by classical theory}}$$

3.4 INFLUENCE OF PRESTRESSING FORCE

To investigate the influence of presstressing force , shell without prestressing force is considered as reference. The stress distribution for the same shell of example 1. of article 2.6 is shown in fig. 3.1. It is found that the agreement of N_x and $N_{x\phi}$ obtained by beam and shell theory is close enough but for N_ϕ and M_ϕ it differs. The contribution of P_e and external loads along with the dead load of shell towards σ_x is shown from fig. 3.2 it is observed that contribution of P_e, calculated by exact theory, is more prominent near the region of shell edge & edge beam and less significant towards crown* while it's

---

* and it's variation is highly nonlinear along the depth of the transverse section of the shell.

influence follows linear path for beam theory. Both the method give contribution of P_e as tensile stress at crown and compressive at bottom. The influence of P_e, evaluated by beam method, at the crown is 7.89 times greater than that, calculated by exact method while for the bottom of edge beam the ratio is 1.81.

3.5 EFFECT OF BOUNDARY CONDITIONS

The boundery conditions and the deformations of the shell, with and without prestressing force, determined both by beam and classical theory are given in the table 3.1

Table 3.1 DEFORMATIONS OF SHELL EDGE

Boundery condition for classical theory	Assumption of beam theory	Deformations			
		P_e=400.000 kg		P_e = 0	
		Classical	Beam	Classical	Beam
$w_{vs} = w_{vb}$	$w_{vs} = w_{vb}$ (assumption 1 of section 1.7(a))	w_v = -48 mm	3.5m.m	21 mm	15.6
H = 0 (w_h exists)	w_h=negligible (assumption 2)	w_h = -35 m.m.	negligible	1 mm	negligible
M = 0 (w_m exists)	w_m= negligible	w_r=-0.077 (rad)	negligible	0	negligible

For $P_e = 0$, $\delta_{vn} = 0.0476$ and $\delta_{vm} = 0$. So for the shell without prestressing force w_h and w_m may be assumed as negligible but with prestressing force they can not be assumed as negligible as it is observed in the previous article that for P_e =400,000 $\delta_{vh} = 0.728$ and $\delta_{vm} = 1.9$. Again for

$$P_e = 0, \quad \frac{(w_v) \text{ determined by classical theory}}{(w_n) \text{ determined by beam theory}} = \frac{21.0}{15.6} = 1.345$$

while for no prestressing it is 13.21. Therefore the assumptions for boundary conditions in classical and beam theory may agree when $P_e = 0$ which is not true for P_e = 400,000 kg.

3.6 PARAMETRIC STUDY

The structural behaviour of a cylindrical shell is very sensitive to the effect of certain parameters. Therefore it is desirable to study the influence of different parameters such as geometry of the shell, magnitude of prestressing force and eccentricities of the cables on the structural behaviours. In this study the effect of parameters considered are,

1. Span to radius ratio, L/R
2. Semi central angle, ϕ_k
3. Depth of edge beam, d
4. Prestressing force, P_e
5. Ecentricities of the cable, e_o and e_m

Variation of one parameter is affected at a time while others are kept constants to observe it's influence on the stress resultants and the deformations.

As the arch analysis is dependent on the beam analysis and if beam analysis yields accurate enough longitudinal and shear stress resultants, then M_ϕ and N_ϕ could be determined approximately correct by proper arch analysis satisfying the boundary conditions. Therefore paramatric study is done for σ_x, $N_{x\phi}$ and defromations. In the figures the variation of $N_{x\phi}$ is shown where longitudinal stress calculated by beam theory is close enough with that determined by classical theory.

The non-dimensionalised parameters which are considered for the study are given in table 3.2

Table 3.2 NON DIMENSIONALISED PARAMETERS

Shell parameters	Non Dimensionalised form	Expression
Semi-central angle	ϕ_k	ϕ_k
Radious, R	Span to radious ratio	L/R
Depth of edge beam, d	Depth to span ratio	d/L
Prestressing force P_e	Coefficient of prestressing force, p	$p = \dfrac{P_e}{q_{1c}\, L^2}$
Eccentricities	End eccentricity to depth ratio	e_O/d
	Mid eccentricity to depth ratio	e_m/d

$q_{1c} = q_u + q_g\, \phi_k / \sin \phi_k$

i) σ_x : Longitudinal stress, σ_x at the crown decreases as ϕ_k increases from 30° to 45° in all L/R ratios but increases as L/R increases for all values of ϕ_k for both p=0.4 and p=0. At the bottom of the edge beam the behaviour is just reverse. Both beam and classical theory yields the above behaviour beam theory gives the value of σ_x at crown lower than that of classical theory for all values of L/R and ϕ_k when p=0.4 but for p=0 it always gives higher value than classical theory. At the bottom of the edge beam, beam theory allways underestimates the value of σ_x for p=0.4 but for p=0 it gives higher value than classical theory for L/R=5 and ϕ_k = 30, L/R = 10 and ϕ_k = 30° and 40°. Rest of the cases it gives lower value than the classical theory. The agreement in the variation along the depth of the transverse section does not achieve for p = 0.4. When p = 0 the agreement is there and beam theory gives the varation of σ_x accurate enough except the two cases of L/R = 3.33 when ϕ_k = 40° and 45°. σ_x attains it's minimum value in between the shell edge and ϕ = 10° for all the cases except for ϕ_k = 30° and 40° when L/R = 10 and maximum value at the bottom of the edge beam by classical theory when p = 0.4. For the same value of p beam theory gives minimum value at the crown and maximum value at the bottom. σ_x is of compressive nature by both the theory for p = 0.4 all along the depth. But for p = 0 maximum compressive stress occurs at the crown and maximum tension occurs at the

bottom of the edge beam by both beam and classical theory.

ii) $N_{x\phi}$: For non prestressing stress there is good agreement between beam and classical theory in the distribution of $N_{x\phi}$. The minimum error is 3.5% while the magnitude of maximum error is 11%. For prestressing shell no comprasion exists in between beam and classical theories. For reinforced shell error increases when smaller values of ϕ_k has been taken for **higher** value of L/R.

b) Deformations

i) Vertical deformations: When p = 0.4 the vertical deformation calculated by classical theory, gives the magnitudes much higher than those determined by beam theory. By beam theory difference between the vertical deformations for different values of ϕ_k is not significant where for classical theory it is opposite. For p = 0 mode of the variation is more or less same and the difference of magnitude is not much.

ii) Horizontal deformations: Both for p = 0 and p = 0.4 w_h can not be determined by beam theory while classical theory gives significant magnitude for both the cases.

iii) Rotational deformations: Mode of variation for p = 0 is similar with that of w_h. Part when p = 0.4 it is to some extent different. Classical theory gives significant magnitudes for both p = 0 and p = 0.4 while beam theory neglects it's magnitudes.

The variation of w_v, w_h and w_m with L/R and ϕ_k are shown in fig. 3.10.

3.8 EFFECT OF THE DEPTH OF EDGE BEAM

To study the effect of the edge beam on the structural behaviour the values of d/L ratio consideration are 0.05, 0.04, 0.03, 0.02. Other parameters are kept constant, as given,

$$L/R = 0.03,\ \phi_k = 30^\circ,\ R/t = 100,\ b/t = 1.25,\ p = 0.4 \text{ and } 0,$$
$$e_0/d = e_m/d = 0.4$$

a) Stress Resultants

The behavior of the stress resultants are shown in figure 3.5 and 3.6 and the following observations are made:

i) σ_x : When p = 0 the agreement between the beam and the shell theory in the variation is very much dominant for all the values of d/L. When d = 0.4 compression occurs both at the top and bottom by classical and beam theory where σ_x at crown has lower value than at the bottom of the edge beam for a value of d/L 0.5 to 0.4 classical theory gives tension at the edge of the shell at d/L = 0.5 and it's magnitude decreases as d/L decreases and the shell is totally in compressive state when d/L = 0.3. Beam theory does not give any where tension. For d/L =0.02 the value of σ_x at the top is always less than at the bottom and shell is fully in compressive state by both the theories. The agreement in the variation is slightly achieved as d/L decreases.

ii) $N_{x\phi}$: No agreement between beam and classical theories was found for prestressed shell. For reinforced concrete shell beam theory gives accurate enough stresses which is well comparable with classical theory.

b) Deformations

Variation of deformations with d/L are given in fig.3.10. From the study of this curve following observations are made.

i) Vertical deformation (w_v): For p = 0 at d/L = 0.02 beam theory gives 0.722 time vertical deformation determined by classical theory but at d/L = 0.05 beam theory yields 1.49 times greater defromation than the classical theory. When p = 0.4, deformation by beam theory is always less than the that determined by classical theory. As d/L increases defference between the defromation is increased. Classical theory always gives negative deformation but for beam theory it is negative for the range d/L = 0.04 to 0.05. W_v decreases for both p = 0 and p = 0.4 and magnitude for p = 0 is always greater than the magnitude at p = 0.04 by both the theories.

ii) Horizontal deformation (w_h) : By beam theory horizontal deformation is negligible and it's magnitude can not be determined. w_h determined by classical theory, for p = 0.04 is always negative and the magnitude increases as d/L increases. For p = 0 w_h is possitive and decreases d/L increases. At d/L=0.0325 it is 0 and for other values it is negative.

iii) Rotational deformation (w_m) : The variation of w_m with d/L is of similar type as that of w_h.

3.9 EFFECT OF ECCENTRICITY:

The following combination of eccentricities are considered to study the effect of eccenticities in structural behavior.

1. $e_0/d = 0$, $e_m/d = 0$
2. $e_0/d = 0$, $e_m/d = 0.4$
3. $e_0/d = 0.4$, $e_m/d = 0$
4. $e_0/d = 0.4$, $e_m/d = 0.4$
5. $e_0/d = 0.2$, $e_m/d = 0.4$
6. $e_0/d = 0.4$, $e_m/d = 0.2$

other parameters are kept constants as follows

$L/R = 3.33$, $d/L = 0.03$, $R/t = 100$, $b/t = 1.25$, $p = 0.4$

From thestudy of the behaviour curve for the stress resultants following observations are made.

i) σ_x: For the combination 3 the agreement between beam and classical method is close. For combination 1 defference between the σ_x determined by beam theory and classical theory is not much. Combination 6 exibits more-or less same manner of variations for combinations 2,4 and 5 no agreement exists. Both the theories keep the shell in the compressive state through out the transverse depth. Beam theory gives lower values of σ_x at the crown and bottom of edge beam than the values determined by classical theory. But σ_x determined inthe region of mid depth by classical theory. has lower values than those determined by beam theory. The

influence of eccentricities on σ_x at the crown have little while at the bottom of the edge beam they have significant influence.

ii) $N_{x\phi}$: For all the combinations, except 1, the difference between $N_{x\phi}$ determined by the classical theory and the beam theory decreases as e_m/d increases and e_0/d decreases. both the theories gives lower values of $N_{x\phi}$ when $e_m/d = 0.4$ and for the values of e_0/d greater than the e_m/d, both the theories gives higher values of $N_{x\phi}$. Eccentricities have very sensetive influence on the magnitude of $N_{x\phi}$ for both the theories.

3.10 EFFECT OF PRESTRESSING FORCE:

Values of p considered for this study are 0.8, 0.6 , 0.4, 0.2 and and other parameters are considered as follow: $L/R=3.33$ $\phi_k = 30^0$, $R/t - 100$, $b/t = 1.25$, $d/L = 0.03$.

a) Stress Resultants:

The behaviour curve for stress resultants are shown in Figure 3.8. From the study following observations are made.

i) σ_x : For $p = 0.8$ beam theory give σ_x at the crown as tension while for classical theory it is compresion for $p = 0.2$ and 0 both the theories give compression at the top and tension at the bottom. For other three values of p σ_x at bottom is compression and the magnitude decreasing with

when p is decreasing. From both the theory it is observed σ_x at crown increases as p decreases. The agreement between beam theory and classical theory is close for p = 0.2 and **0.**

ii) $N_{x\phi}$:

The differences between the magnitude obtained by beam and classical theory decreases as p decreases and close agreement achieved for p = 0.2 and 0.

b) <u>Deformations:</u>

The behaviour curve shown in Figure 3.11.

i) w_v :-

The variation of w_v with p is linear for both the case. Beam theory gives the magnitude always less except when p = 0.2. For p = 0.4 w_v has very small magnitude in posetive direction by beam theory while by classical theory its magnitude is about 8 **times** greater in opposite direction.

ii) w_h :-

The variation is linear. At p = 0 it is magnitude is not 0 but very smalland attains maximum magnitude at p = 0.8 by classical theory while beam theory assume it as negligible.

iii) w_m:-

Same type of variation as that of w_h only difference is it it is magnitude at p=0 is 0.

3.14 DISCUSSION:

From the above investigations following observations are made,

i) The discrepency between the boundary conditions and the assumptions of beam theory is more significant for prestressed shell than non prestressed shell.

ii) Beam theory can be applicable for all values of L/R , d/L and ϕ_k when p = 0 .

iii) Beam theory may be applicable for a limited range of shell parameters for prestressed shell.

iv) Restraints, fully or partially, yields the stress resultants which compensates each other for w_m, w_h

In the next chapter on the basis of the observations, made from the above investigations, theory for arch analysis developed using casting liano's second theorem taking consideration the variation of loads with Sin $\frac{n \pi x}{l}$.

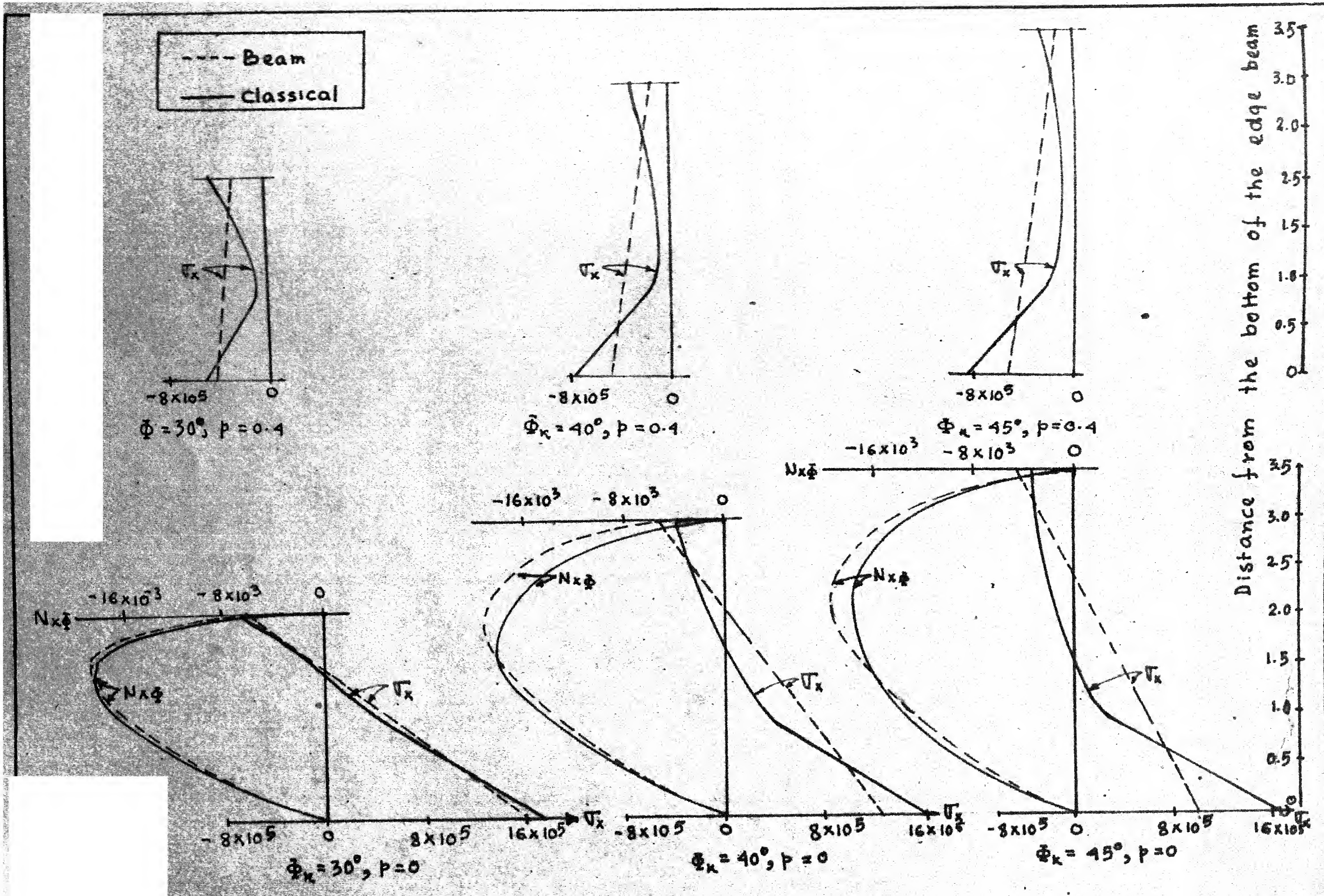


Fig. 3.4. VARIATION OF STRESS RESULTANTS FOR L/R = 3.33

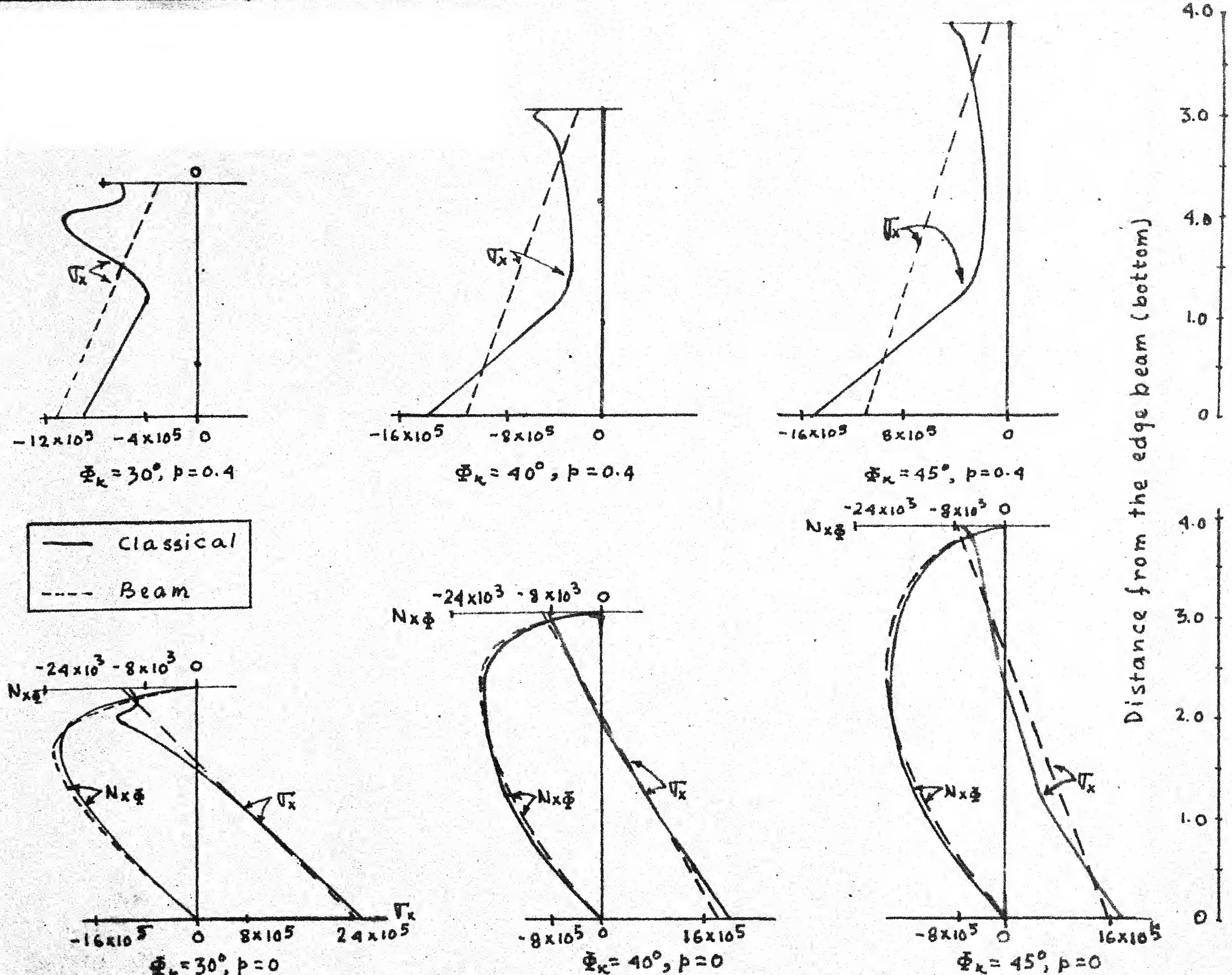


Fig. 3.5. VARIATION OF STRESS RESULTANTS FOR $L/R = 5.0$

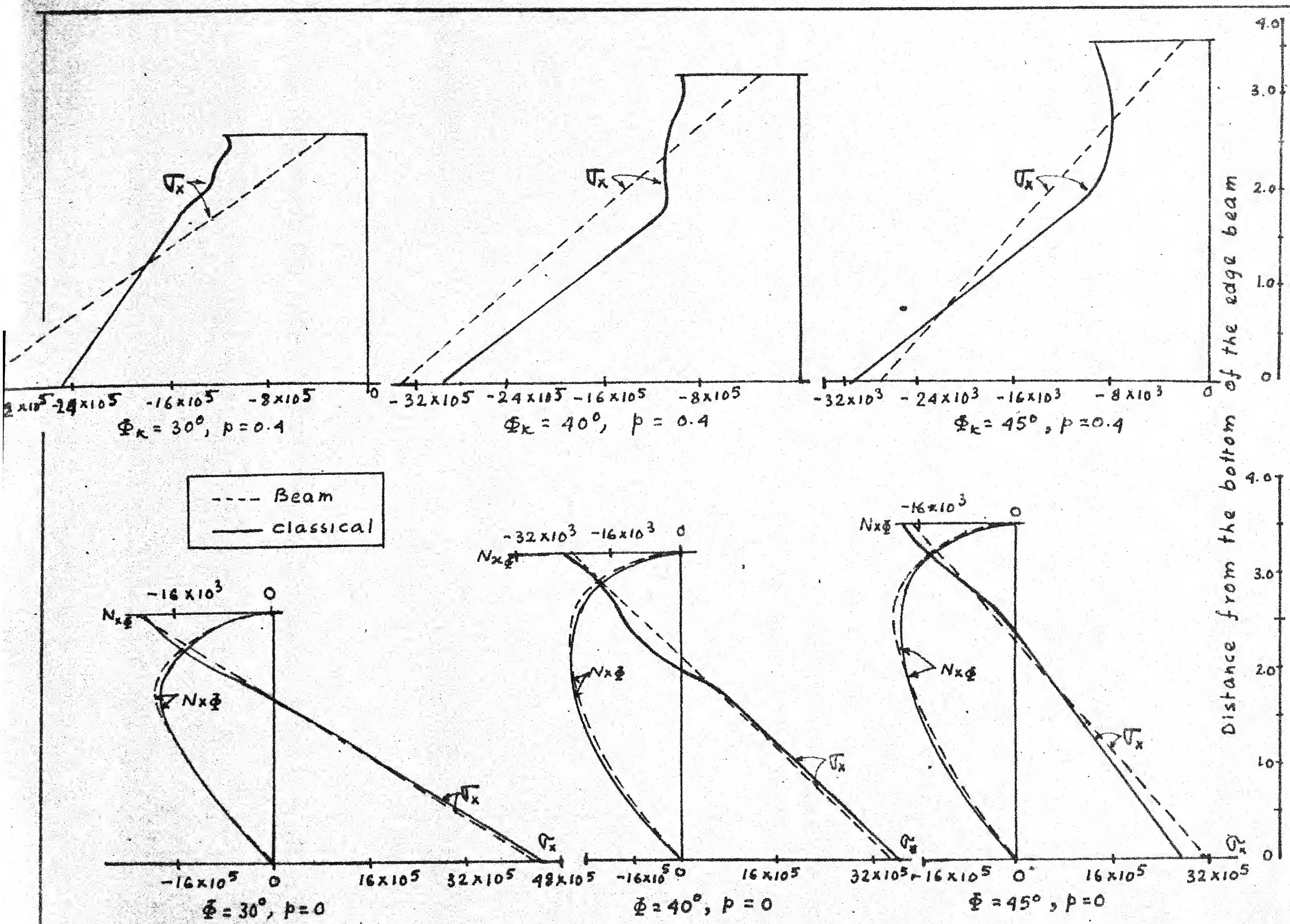


Fig. 3.6 VARIATION OF STRESS RESULTANTS FOR L/R = 10

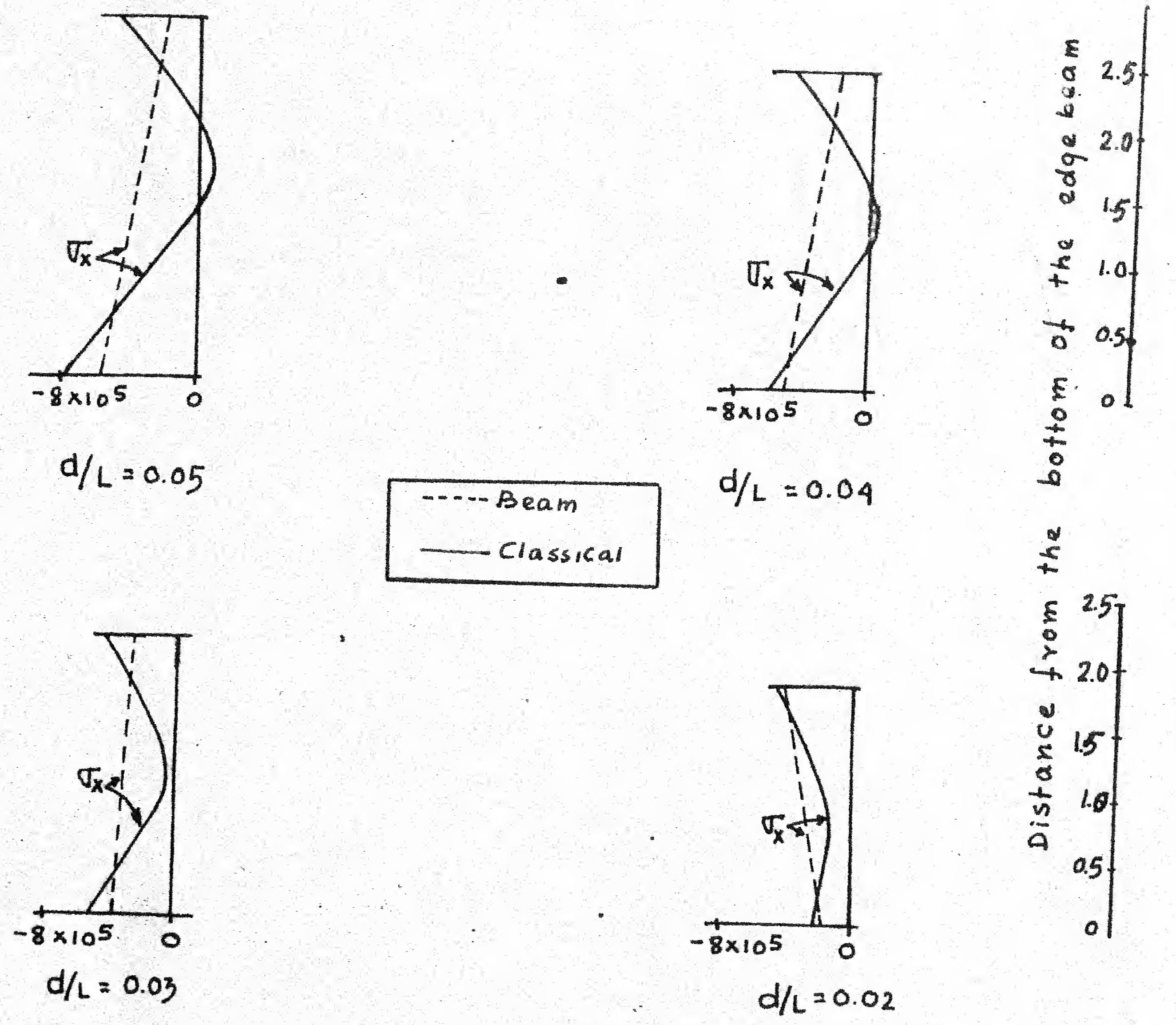


Fig. 3.7 VARIATION OF STRESS RESULTANTS WITH D d/L, p=0.4

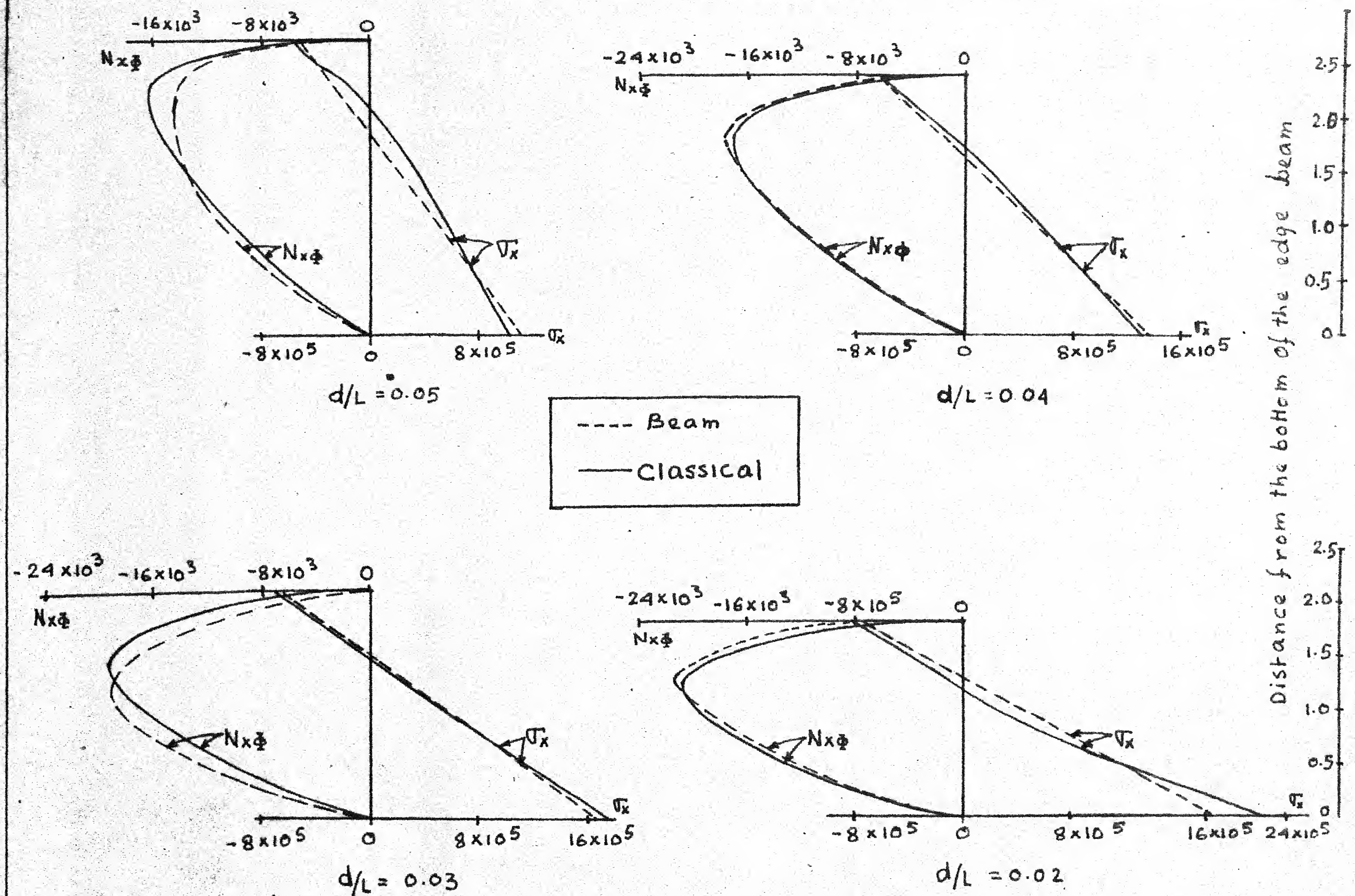


Fig. 3.8 VARIATION OF STRESS RESULTANT WITH d/L AND $p=0$

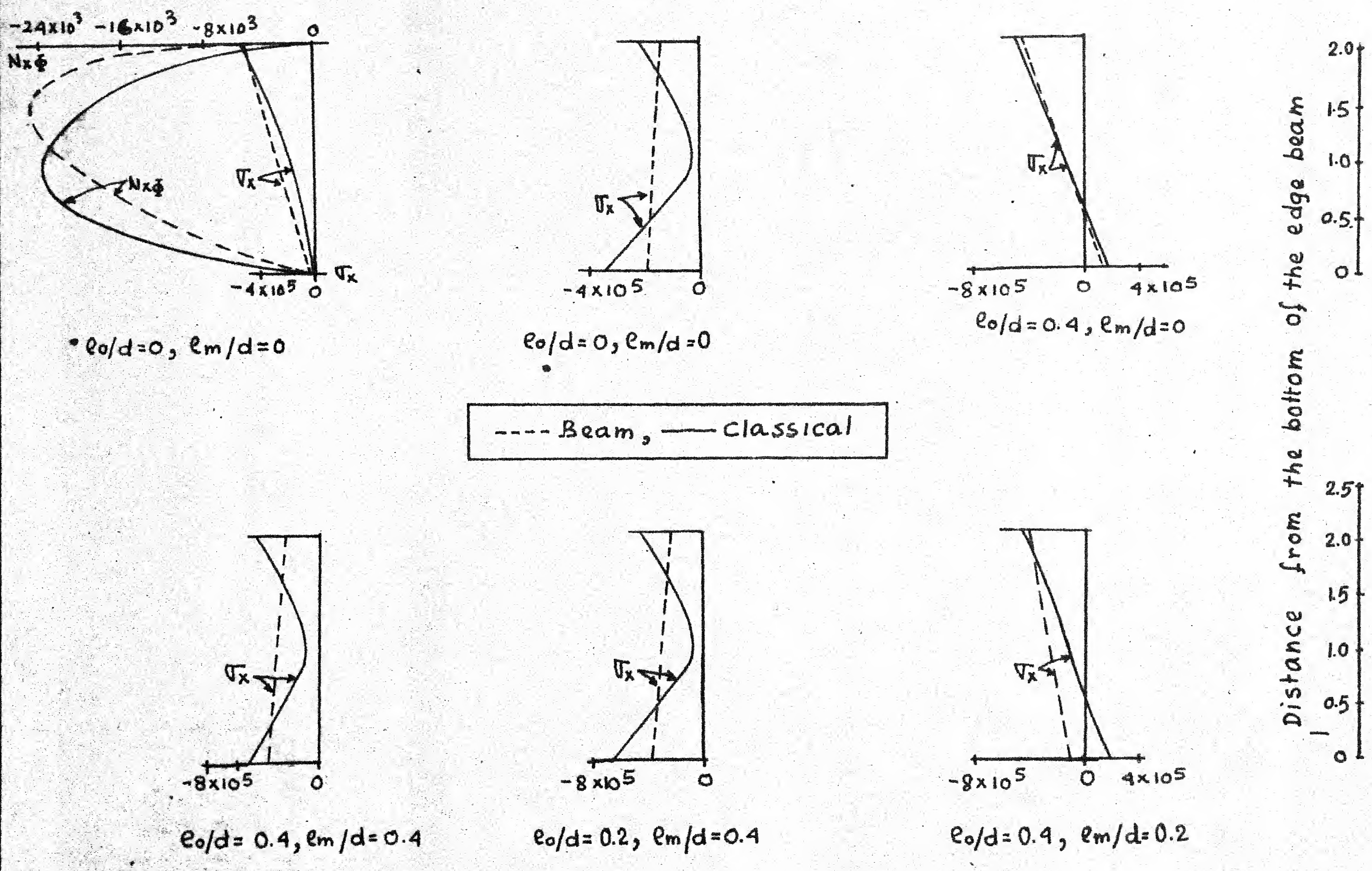


Fig. 3.9 **VARIATION OF STRESS RESULTANTWITH e_o/d, e_m/d**

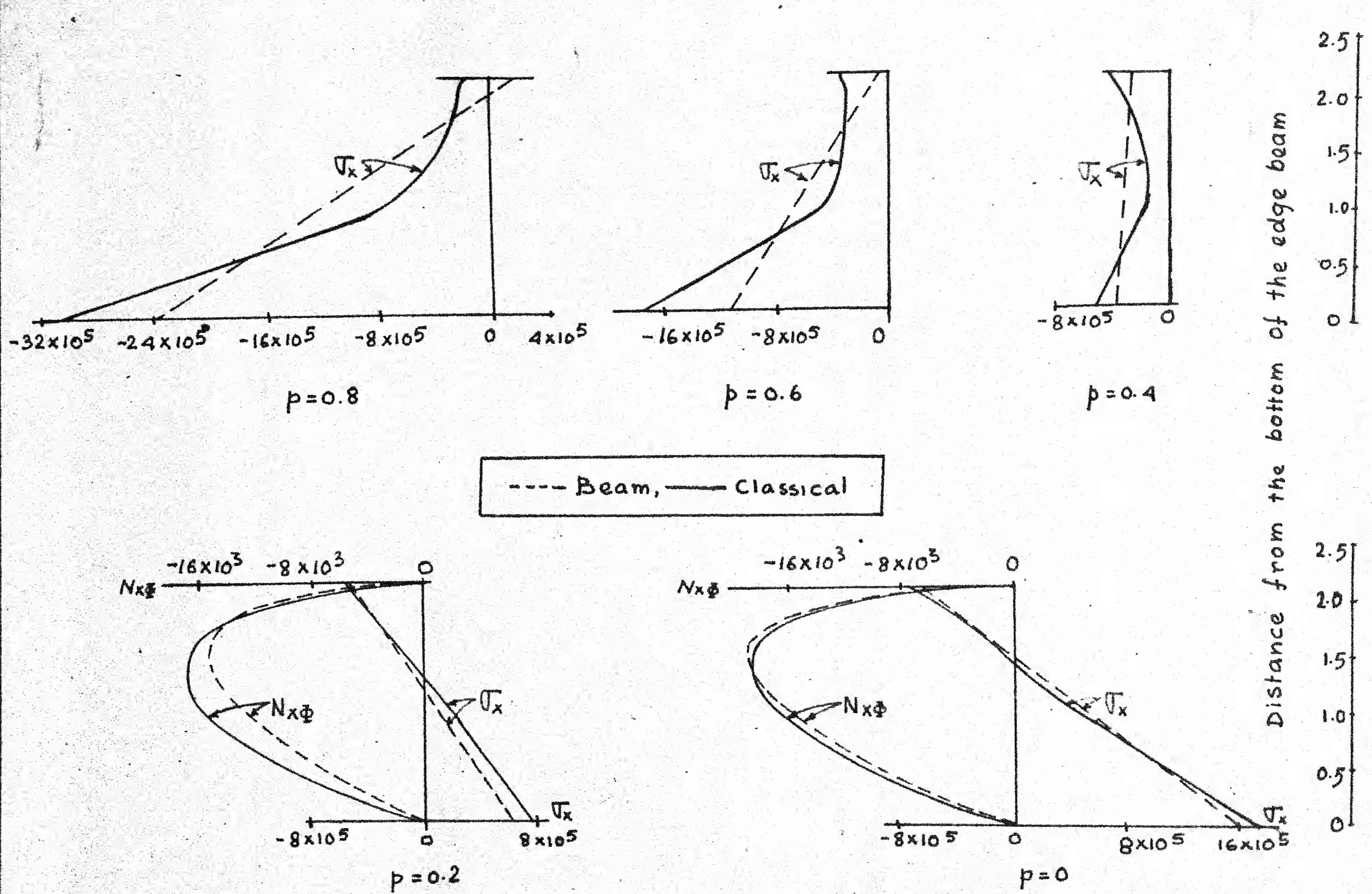


Fig. 3.10. VARIATION OF STRESS RESULTANTS WITH p

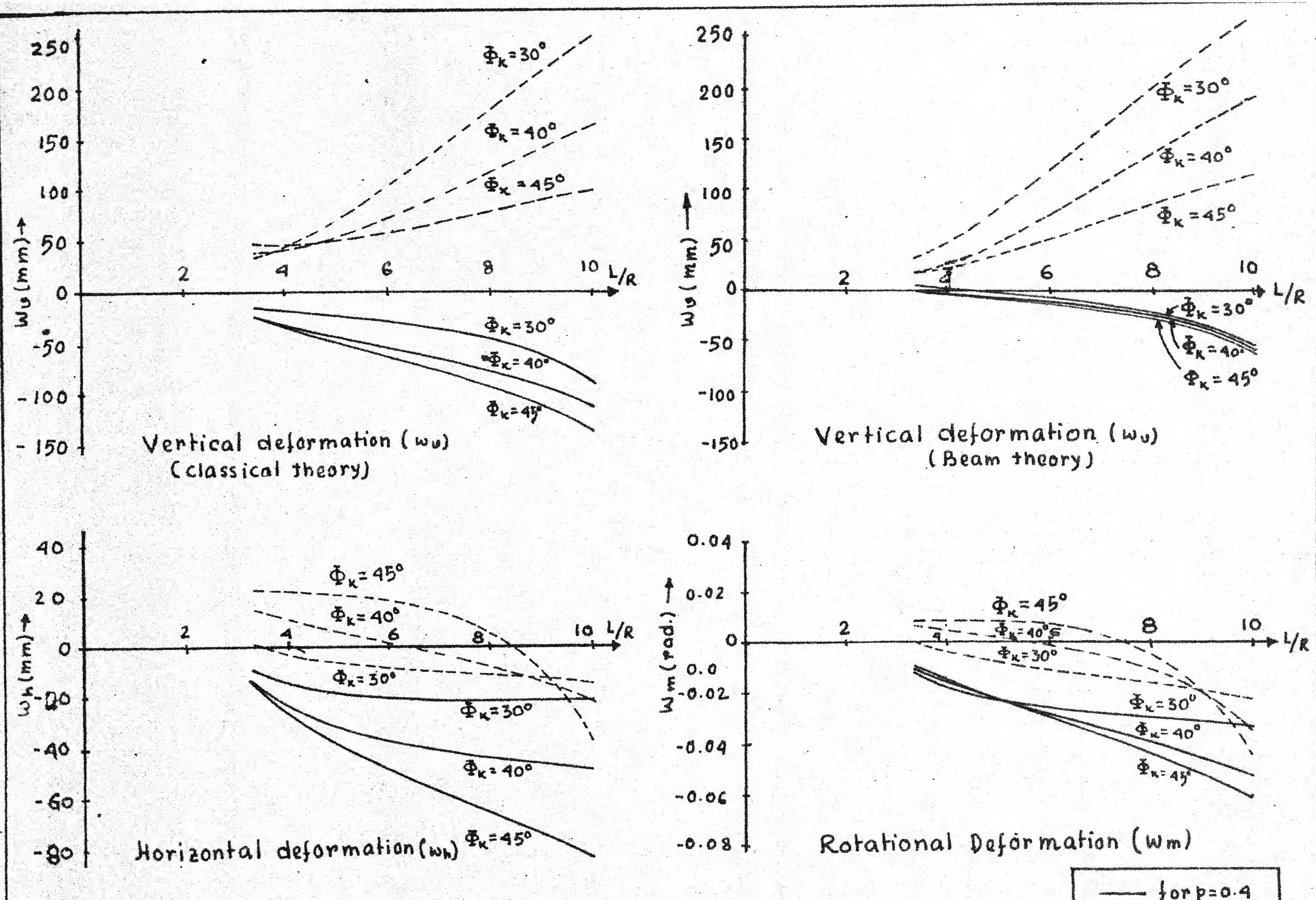


Fig. 3.11 VARIATION OF DEFORMATIONS WITH L/R.

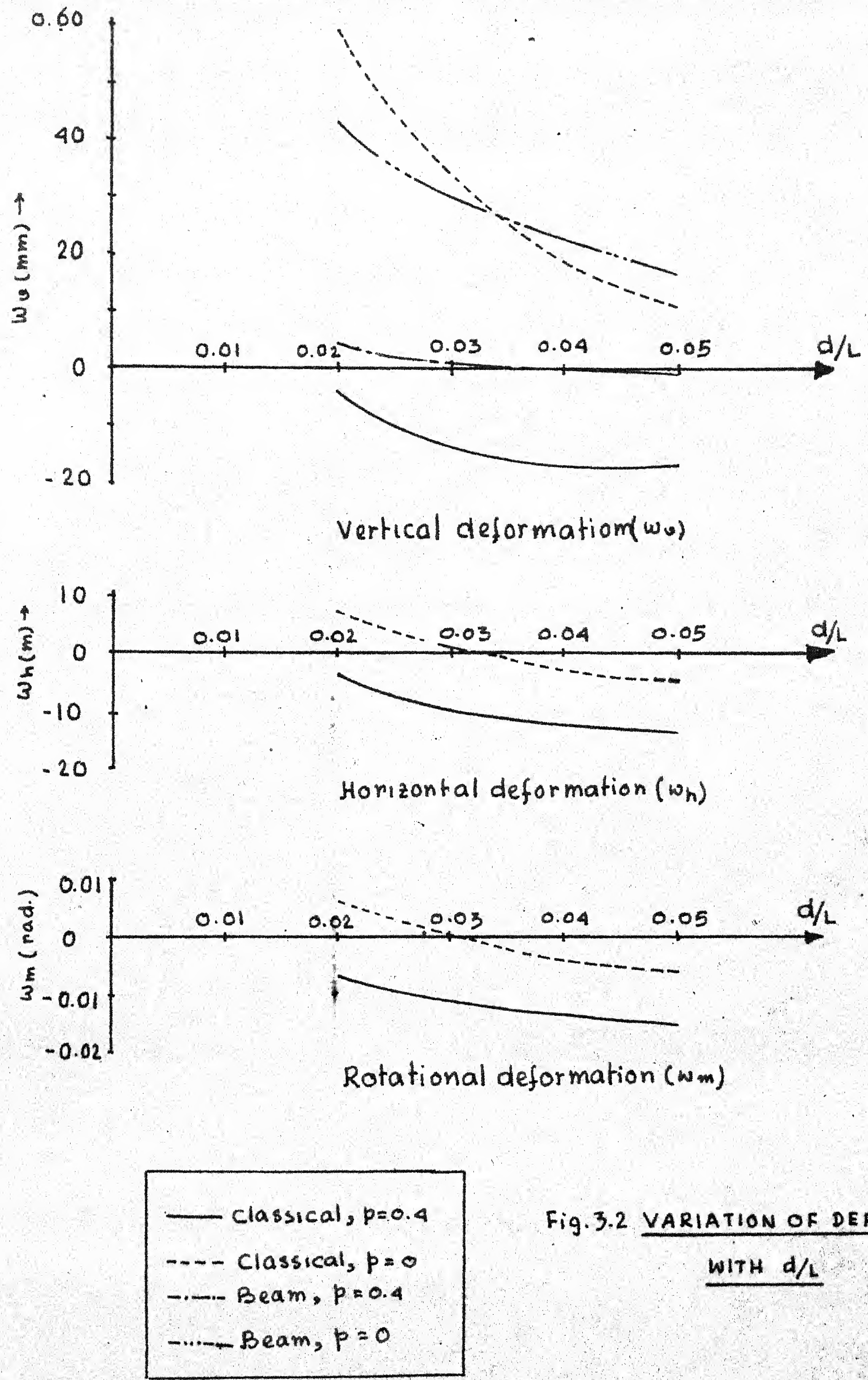


Fig. 3.2 VARIATION OF DEFORMATION WITH d/L

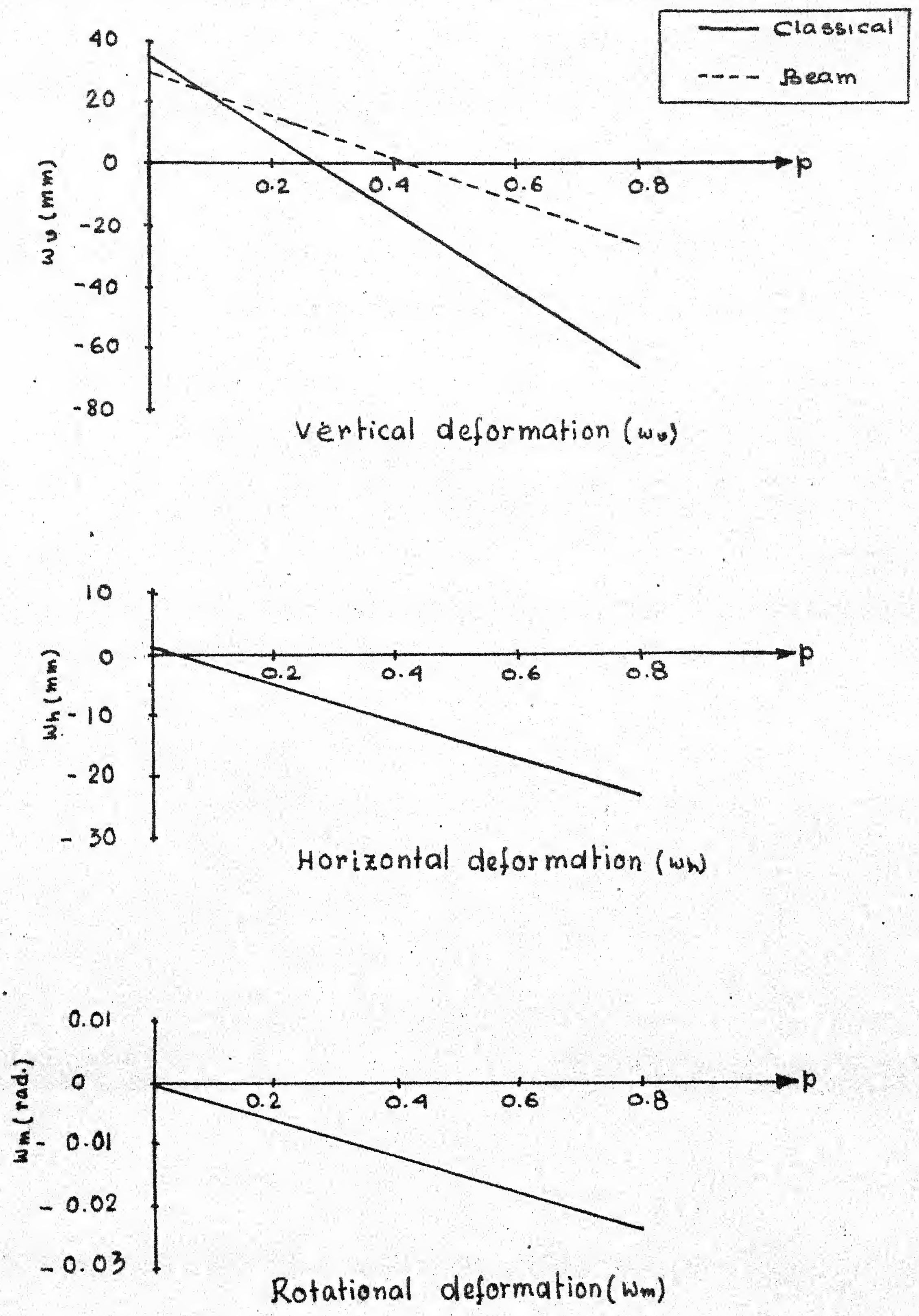


Fig. 3.13 VARIATION OF DEFORMATIONS WITH p

CHAPTER-IV

DESIGN OF CYLINDRICAL SHELL BY SIMPLIFIED BEAM AND ARCH ANALYSIS

4.1 INTRODUCTION:

It is evident from Figs. 3.3 , 3.4, 3.5 and 3.7 that beam theory yields very good approximate stress distributions for N_x and $N_{x\phi}$ for almost all combinations of L/R , ϕ_k and d/L for non prestressed shell. But in article 3.4 it was found that the distributions for N_ϕ and M_ϕ by beam theory for single barrel shell do not agree with classical theory. Again it is a well established fact that beam theory yields accurate enough stress distributions for N_ϕ $N_{x\phi}$, N_ϕ and M_ϕ for the interior shell of multibarrel shell group with and without edge beam when p=0. The object of this chapter is to establish such a formulation by which any type of cylindrical shell can be analysed and designed by beam and arch analysis.

Like conventional beam method this method has two distinct parts, beam analysis and arch analysis. The beam analysis does not change but the arch analysis is different. The theory of proposed method is developed assuming the loading on the shell varies with Sin $n\pi x/L$.

In addition to the assumptions made in article 1.7, the method assumes, "The principle of superposition holds, in which case the strain energy is a homogenious function

of second degree, where the strain energy can be expressed as a function of external forces". This assumption is mainly for arch analysis.

4.2 BEAM ANALYSIS

The beam analysis is all most similar to that which was described in article 1.7 (b). The loading is assumed to vary along the longitudinal direction as half sine series.

Loading on shell $q = q_0 \operatorname{Sin} mx$

Where $q_0 = 1_c \left(q_u + \frac{\phi_k}{\operatorname{Sin}\phi} q_g \right)$

Moment at any longitudinal section, $M = \frac{q_0}{m^2} \operatorname{Sin} mx$

Shear force at any section, $S_f = \frac{1}{m} q_0 \operatorname{Cos} mx$ C.G., area and moment of inertia of, the sections are determined as before, only Q_i at any node i determined as follow,

$$Q_i = 2 R\phi_i t \; (Z_s - R\, k_{zi})$$

$\therefore$ At any longitudinal section, $\sigma_x = \frac{Z_{yt}}{I_s} \frac{1}{m^2} q_0 \operatorname{Sin} mx$ (4.1)

At any longitudinal section, $N_{x\phi} = \frac{Q_i}{2\ I_s} \frac{1}{m} q_0 \operatorname{Cos} mx$ (4.2)

Specific shear, $\frac{\partial N_{x\phi}}{\partial x} = - \frac{Q_i}{2\ I_s} q_0 \operatorname{Sin} mx$ (4.3)

4.3 ANALYSIS OF CIRCULAR ARCH RESTING ON ELASTIC FOUNDATION

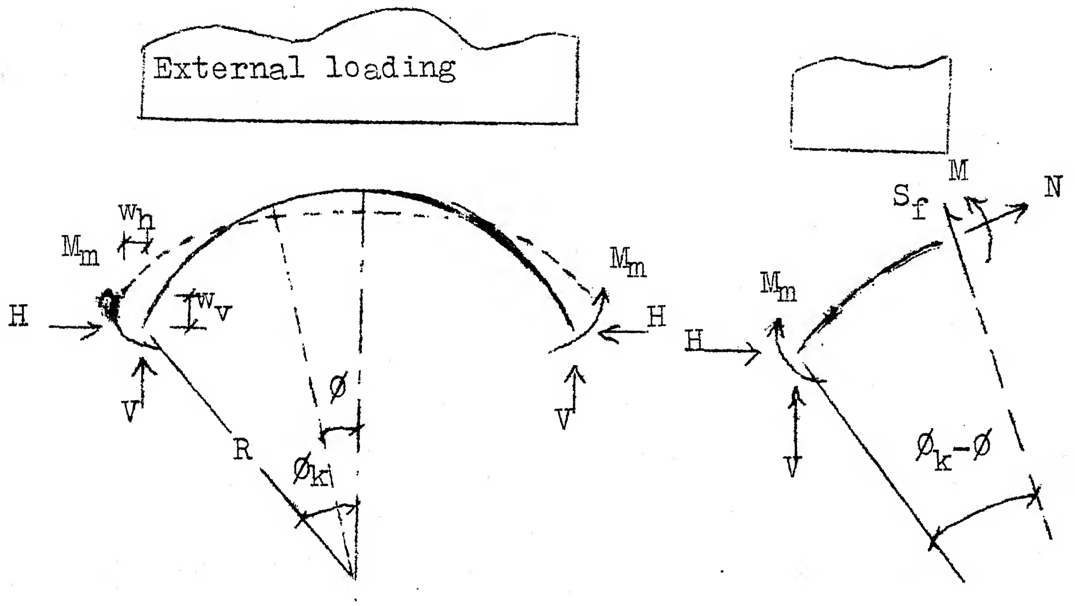


Fig. 4.1 CIRCULAR ARCH ON ELASTIC FOUNDATION

Moment at any section ϕ,

$$M = VR(\sin\phi_k - \sin\phi) + HR(\cos\phi_k - \cos\phi) + M_s + M_m \tag{4.4}$$

$$N = -V\sin\phi - H\cos\phi + N_s \tag{4.5}$$

$$S_f = -V\cos\phi - H\sin\phi + S_{fs} \tag{4.6}$$

Where, M_s = Moment due to external loading

M_m = Elastic redundant moment at support

N_s = Axial force due to external load

Total strain energy of the arch,

$$U = \int_0^S \frac{M^2}{2EI}\, ds + \int_0^S \frac{N^2}{2AE}\, ds$$

From Castigliano's, second theorem,

Vertical deformation at the support, $w_{v1} = \frac{\partial U}{\partial V}$ (4.7 a)

Horizontal deformation at the support $w_{h1} = \frac{\partial U}{\partial H}$ (4.7 b)

Rotational deformation at the support, $w_{m1} = \frac{\partial U}{\partial M_m}$ (4.7 c)

Putting eqns. 4.4 and 4.5 on eqs. of 4.6, and equating w_{v1}, w_{h1}, w_{m1} to w_v, w_h and w_m,

$$(T^3 k_{mv} + T k_{nv})\, V + (T^3 k_{mvh})\, H + (T^3 k_{mvm})\, M' = Ew_v - T^3 K_{11} \quad (4.8)$$

$$(T^3 k_{mvh})\, V + (T^3 k_{mh} + T k_{nh})\, H + (T^3 k_{mhm})\, M' = Ew_h - T^3 K_{21} + T K_{22} \quad (4.9)$$

$$(T^3 k_{mvm})\, V + (T^3 k_{mhm})\, H + (T^3 k_{mm})\, M' = Ew_m - T^3 K_{31} \quad (4.10)$$

Where,

$$k_{mv} = 12\phi_k\,(2\,\mathrm{Sin}^2\phi_k + 1) - 6\,\mathrm{Sin}\,2\phi_k \quad (4.11\ a)$$

$$k_{mh} = 12\phi_k\,(2\,\mathrm{Cos}^2\phi_k + 1) - 18\,\mathrm{Sin}\,2\phi_k \quad (4.11\ b)$$

$$k_{mm} = 24\phi_k \quad (4.11\ c)$$

$$k_{mvh} = 12\phi_k\,\mathrm{Sin}\,2\phi_k - 24\,\mathrm{Sin}^2\phi_k \quad (4.11\ d)$$

$$k_{mvm} = 24\phi_k\,\mathrm{Sin}\phi_k \quad (4.11\ e)$$

$$k_{mhm} = 24\phi_k\,\mathrm{Cos}\phi_k - 24\,\mathrm{Sin}\phi_k \quad (4.11\ f)$$

$$k_{nv} = \phi_k - 0.5\,\mathrm{Sin}\,2\phi_k \quad (4.11\ g)$$

$$M' = M_m/R$$

$$k_{nh} = \phi_k + 0.5 \text{ Sin } 2\phi_k \quad (4.11\ h)$$

$$K_{11} = \int_{-\phi_k}^{\phi_k} \frac{M_s}{R} \text{ Sin}\phi_k \, d\phi \quad (4.12\ a)$$

$$K_{21} = \int_{-\phi_k}^{\phi_k} \frac{M_s}{R} (\text{Cos}\phi_k - \text{Cos}\phi) \, d\phi \quad (4.12\ b)$$

$$K_{22} = \int_{-\phi_k}^{\phi_k} N_s \text{ Cos}\phi \, d\phi \quad (4.12\ c)$$

$$K_{31} = \int_{-\phi_k}^{\phi_k} \frac{M_s}{R} \, d\phi \quad (4.12\ d)$$

$$T = \frac{1}{a} \frac{R}{t} \quad (4.13\ a)$$

$$T^3 = \frac{1}{a} \left(\frac{R}{t}\right)^3 \quad (4.13\ b)$$

a = width of the radial sectionof the arch

t = depth of the radial section of the arch

Eqns. 4.7, 4.8,and 4.9 have 3 un knowns , V, H and M' and hence will give unique solution to determine the values of V,H,M', and putting those values on eqns. 4.4, 4.5 and 4.6 M, N and S_f can be determined.

To derive the eqns. 4.8, 4.9, 4.10 the contribution of S_f to strain energy is neglected and the radial sectional properties of the arch are assumed to be constant with the variation of ϕ. The external loading is also taken symmetric about Z-axis .

4.4 ARCH ANALYSIS

For the arch analysis for the proposed method the elementary strip of the circular arch is considered and this elementary arch strip is considered to be supported on the edge beam. This support is assumd to be elastic support which can deforme in three directions, i.e., vertical, horizontal and rotational directions . In general each support point has three degrees of freedome while actual conditions are determined from the boundary conditions of the shell. The support reactions are determined by satisfying the compatibility of the deformations of shell edge and edge beam at the interface in mid span.

The elementary arch strip is analysed by the method described in article 4.3. The edge beam is assumd to offer restraint against vertical deformation only. Again considering the statical equilibrium the arch is in equilibrium under the action of external loading and specific shear.

As the arch strip in separated, for arch analysis, from edge beam then it would be in equilibrium under the loading of F_v and V

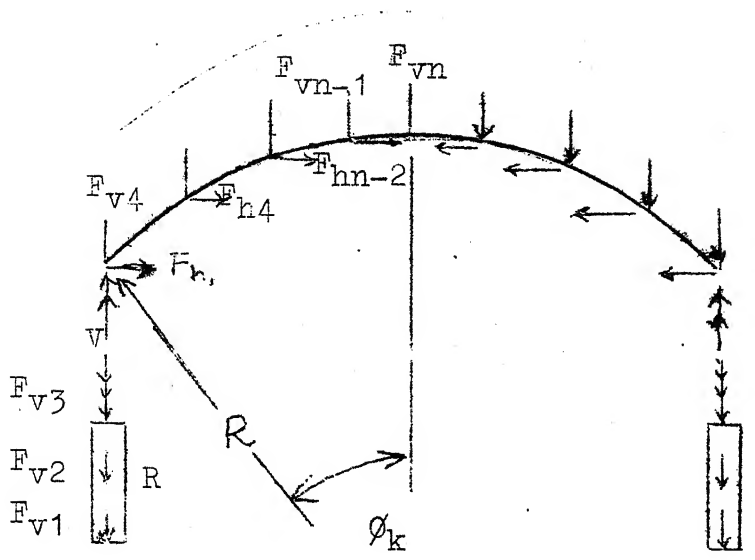


Fig. 4.2 STATICAL EQUILIBRIUM OF ELEMENTARY ARCH STRIP

$\therefore$ For statical equilibrium $\sum_{i=4}^{n} F_{vi} + V = 0$

$$\text{or} \quad V = -\sum_{i=4}^{n} F_v \tag{4.14}$$

Therefore, the arch analysis essentially stands as the solution of two simultaneous eqns. 4.9 and 4.10 satisfying the conditions given by eqn. 4.14. Hence solution may be achieved solving two simultanious Equations. Putting the the value of V from eqns. 4.14 to 4.9 and 4.10

$$(T^3 k_{mh} + Tk_{rh}) H + (T^3 k_{mhm}) M' = E_s w_v - T^3 K_{21} + TK_{22} - (T^3 k_{mvh}) V \tag{4.15}$$

$$(T^3 k_{mhm})H + (T^3 k_{mm})\ M' = E_s w_m - T^3 K_{31} - (T^3 k_{mvm})V \qquad (4.16)$$

Where, $T = R/t$

The unique solution of eqns. 4.15 and 4.16 will determined the values of H and M'. As soon as V, H, M'. are known M_ϕ and N_ϕ at any longitudinal section, are given by:

$$M_\phi = M_s + Rc_{mV}V + Rc_{mH}H + RM', \qquad (4.17)$$

$$M_\phi = N_s + c_{nv}V + c_{nh}H \qquad (4.18)$$

where

$$c_{mv} = \text{Sin}\phi_k - \text{Sin}\phi \qquad (4.19\ a)$$

$$c_{mh} = \text{Cos}\phi_k - \text{Cos}\phi \qquad (4.19\ b)$$

$$c_{nv} = -\text{Sin}\phi \qquad (4.19\ c)$$

$$c_{nh} = -\text{Cos}\phi \qquad (4.19\ d)$$

4.5 BOUNDARY CONDITIONS

For single barrel shell H = 0 and $M' = 0$ as it is assumed that edge beam does not offer any restraints towards horizontal and rotational deformations.

For the interior shell of multibarrel group $w_h = 0$ and $w_m = 0$.

4.6 SUMMARY OF COMPUTATIONS

For the systematic and easy computional work of cylindrical shell analysis, the proposed method is detailed below in 6 steps.

1. Sectional Properties (Ref.Fig.2.3)

a) Assume geometrical properties of the transverse section.

b) $h = R\ (1 - \cos\phi_k) + d$

c) $l_c = 2R\ \sin\phi_k$

d) Area of cross section, $A_c = 2\ Rt\phi_k$

$$A_b = 2\ bd$$

$$A_s = A_c + A_b$$

e) Centre of gravity, $Z_c = R\ k_z$

$$Z_b = h - d/2$$

$$Z_s = (A_c Z_c + A_b Z_b)\ /\ A_s$$

f) Moment of inertia, $I_c = R^3 t\ k_I$

$$I_b = bd^3/6$$

$$Z_{rc} = Z_s - Z_c$$

$$Z_{rb} = Z_b - Z_s$$

$$I_s = I_b + I_c + A_b (Z_{rb})^2 + A_c (Z_{rc})^2$$

g) Co-ordinates of nodal points (Ref. Fig.2.4),

$d\phi = \phi_k/k$

k is always even number such that $d\phi > 5°$

Number of nodal points $n = m+3$, $m=k+1$

$$Z_i = R(1-\cos\phi_i)-Z_s$$

$$Y_i = R\ \sin\phi_i$$

h) Distance of top fiber from C.G., $Z_{yt} = Z_s$

Distance of bottom fiber from C.G., $Z_{yb} = h - Z_s$

i) First moment of area, i) For shell portion

$$Q_i = 2\,Rt\,(Z_s - R\,k_{Zi})$$

ii) For beam portion

$$Q_i = b\,(Z_{yb}^2 - Z_i^2)$$

2. Loading (ref. Fig. 1.1c)

Determine q_u and q_g

$$q_o = \frac{4}{\quad}\left[\left\{\frac{\phi_k}{\sin\phi_k}\,(q_g + \quad t\,) + q_u\right\}\,l_c + 2bd\gamma\right]$$

3. Beam Analysis

a) $\sigma_{xt} = -q_o\,(1/\lambda\ )^2\,(Z_{yt}/I_s)\ \sin mx$

$\sigma_{xb} = q_o\,(1/\lambda\ )^2\,(Z_{yb}/I_b)\ \sin mx$

b) $N_{x\phi_i} = q_o\,(1/\lambda\ )\,(1/2\ I_s)\,Q_i\ \cos mx$

c) Specific shear $= -\,(q_o/2\ I_s)\,\phi_i\ \sin mx$

4. The above computation canbe done systematically by the following tabular form.

Nodal point	ϕ_i in degree	ϕ_i in rad.	Co-ordinates Z_i	Co-ordinates Y_i	σ_x	Q_i	$N_{x\phi}$	Specific shear

5. Arch Analysis

a) Shell | Beam

Shell	Beam
$ds_4 = ds_m = Rd\emptyset/2$, $ds_i = Rd\emptyset$	$ds_1 = ds_3 = d/4$, $ds_2 = d/2$
$dy_i = R\ (\sin\emptyset_{i-1} - \sin\emptyset_i)$	$dy_i = 0$
$dz_i = R\ (\cos\emptyset_i - \cos\emptyset_{i-1})$	$dz_i = ds_i$

where, $\emptyset_4 = \emptyset_k$, $\emptyset_5 = \emptyset_k - d\emptyset/2$

$\emptyset_i = \emptyset_{i-1} - d\emptyset$, $i = 6,7 - - - n-2$

$\emptyset_{n-1} \equiv d\emptyset/2$, $\emptyset_n = 0$

b) $$F_{vi} = q_g\ dy_i + \frac{\partial N_{x\emptyset_i}}{\partial x}\ dz_i$$

$$F_{hi} = \frac{\partial N_{x\emptyset_i}}{\partial x}\ dy_i$$

$$M_{si} = \sum_{j=4,5}^{i} F_{vj}(Y_j - Y_i) + F_{hj}\ (Z_j - Z_i)$$

$$N_{si} = \left(\sum_{j=4,5}^{i} F_{vj}\right) \sin\emptyset_i - \left(\sum_{j=4,5}^{i} F_{hj}\right) \cos\emptyset_i$$

c) The arch analysis should be carried on with the help of above expressioniin the following tabular form.

Nodal points	ds	dy	dz	$\frac{\partial N_{x\emptyset}}{\partial x}$ dz	$q_o dy$	F_v	F_h	M_s	N_s

d) $V = \Sigma F_{vi}$

$$T = R/t$$

$$A_{11} = T^3\, k_{mh} + T\, k_{nh}$$

$$A_{12} = T^3\, k_{nhm}$$

$$A_{21} = A_{12}$$

$$A_{22} = T^3 k_{nm}$$

$$K_{21} = 12 \int_{-\phi_k}^{\phi_k} \frac{M_s}{R} \left(\cos\phi_k - \cos\phi \right) d\phi$$

$$K_{22} = \int_{-\phi_k}^{\phi_k} N_s \cos\phi \, d\phi$$

$$K_{31} = 12 \int_{-\phi_k}^{\phi_k} \frac{M_s}{R} \, d\phi$$

The above integral may be evaluated by Simpson's rule.

$$C_1 = E_s\, w_h - T^3\, K_{21} + T K_{22} - T^3 k_{mvh} V$$

$$C_2 = E_s\, w_m - T^3\, K_{31} - T^3\, k_{mvm} V$$

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} H \\ M' \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \tag{4.20}$$

H and M' is solved putting boundery conditions on 4.20.

6. Final values of M_ϕ, N_ϕ

$$M_\phi = M_s + R c_{mv} V + R c_{mh} H + R M'$$

$$N_\phi = N_s + c_{nv} V + c_{nh} H$$

The values of k_I, k_x, k_{mv}, k_{mh}, k_{mm}, k_{mvh}, k_{mvm}, k_{mhm}, k_{mv}, k_{nh} are given in appendix for different values of ϕ and ϕ_k.

4.7 ILLUSTRATED EXAMPLE

Example - 2

Span = 30m, ϕ_k = 30°, R = 9m, d=90cm, b =11.25 cm, t=9cm.

Loading:	Lime terracine	= 45	kg/m^2
	Live load	= 75	kg/m^2
	Total	= 120	kg/m^2

Step-1

Sectional properties:

$h = 2.106$ m, $l_c = 9.0$ m, $A_s = 1.0507\ m^2$, $Z_s = 0.6466$ m

$I_s = 0.3795\ m^4$

$d\phi = 5°$ where k = 6, m = 7, n = 10

$Z_{yt} = 0.6466$ m, $Z_{yb} = 1.4594$ m

In shell portion $Q_i = 1.62\ (0.6466 - 9\ k_z)$

In beam portion $Q_i = 0.1125\ (2.12 - Z_i^2)$

Step-2

$$q_o = \frac{4}{\pi}\left(\frac{0.5236}{0.5} \times 3652.656 + 2 \times 0.9 \times 0.1125 \times 2400\right) = 4650.7061\ kg/m$$

Step-3

Longitudinal stress at top fiber, $\sigma_{xt} = -717000$ kg/m

Longitudinal stress at bottom fiber, $\sigma_{xb} = 1620000$ kg/m

$N_{x\phi_i} = 58475.1435\ Q_i$

Specific shear = 6127.4165 Q_i

Step-4

Table 4.1 STRESS RESULTANTS σ_x, $N_{x\phi}$

Nodal point	ϕ In degree	ϕ In radian	Co-ordinates Z_i (m)	Co-ordinates Y_i (m)	Q (m^3)	σ_x (kg/m^2)	$N_{x\phi}$ (kg/m)	Specific shear (kg/m^2)
1			1.4594	4.5	0	1620000	0	0
2			1.0094	4.5	0.1250		7310.9125	765.12
3			0.5594	4.5	0.2044		11950.8231	1251.79
4	30	0.5236	0.5594	4.5	0.2044		11950.1215	1251.79
5	25	0.4363	0.1966	3.8036	0.2571		15020.3541	1574.53
6	20	0.3490	-0.1018	3.0782	0.2629		15380.3618	1610.05
7	15	0.2618	-0.3399	2.3294	0.2308		13500.7451	1413.46
8	10	0.1745	-0.5099	1.5629	0.1699		9920.4175	1040.50
9	5	0.0872	-0.6123	0.7844	0.0897		5250.3950	549.95
10	0	0	-0.6466	0	0	-717000	0	0

Step - 5

Table 4.2 CALCULATION OF M_s AND N_s

Nodal point	ds	dy	dz	$\frac{\partial N_{x\phi}}{\partial x}$ dz	$q_g ds$	$q_u dy$	F_v	F_h	M_s	N_s
1	0.225	0	0.225	0	77.235	0	77.235	0	0	-
2	0.45	0	0.45	-345.2121	155.45	0	-189.7621	0	0	-
3	0.225	0	0.225	-284.1953	77.235	0	-206.9603	0	0	-
4	0.3927	0.3443	0.1839	-236.4631	167.9952	0	- 68.4679	430.9913	0	- 305.8734
5	0.7854	0.7116	0.3318	-523.4311	335.9904	0	-187.4407	1120.44	- 180.5531	-1298.3477
6	0.7854	0.7378	0.2685	-432.8003	335.9904	0	- 96.8099	1187.8999	- 389.8418	-2453.9975
7	0.7854	0.7584	0.2032	-286.7170	335.9904	0	48.2734	1071.9752	- 742.7895	-3681.4553
8	0.7854	0.7732	0.1363	-141.8208	335.9904	0	194.1696	761.6494	-1152.2685	-4503.4460
9	0.7854	0.7822	0.0684	- 36.6169	335.9904	0	299.3735	430.1746	-1598.1713	-4914.8235
10	0.3927	0.3915	0.02	0	167.9952	0	167.9952	0	-2013.2314	-4933.6206

$$V = \sum_{j=4}^{10} F_{vj} = 356.0932$$

The shell is single barrel, so, H = 0 and $M' = 0$

Step - 6

Final stress resultant is calculated and given in table 4.3 and companed with the result calculated by classical theory.

Table 4.3 M_ϕ AND N_ϕ

ϕ	c_{mv}	M_ϕ 1	M_ϕ 2	c_{nv}	N_ϕ 1	N_ϕ 2
30	0	0	0	-0.5	- 484	- 180
20	0.15798	121	137	-0.34202	-2583	-2270
10	0.32635	-110	- 96	-0.17365	-4565	-4257
0	0.5	-424	-460	0	-4934	-5010

Column 1 and 2 of M_ϕ and N_ϕ stands for the values calculated by proposed method and classical theory respectivty. The variation of stress resultants are shown in figure 4.3.

4.8 DESIGNOOF REINFORCEMENTS

a) Theoretical Aspects

The 4 main stress resultants, for the purpose of design

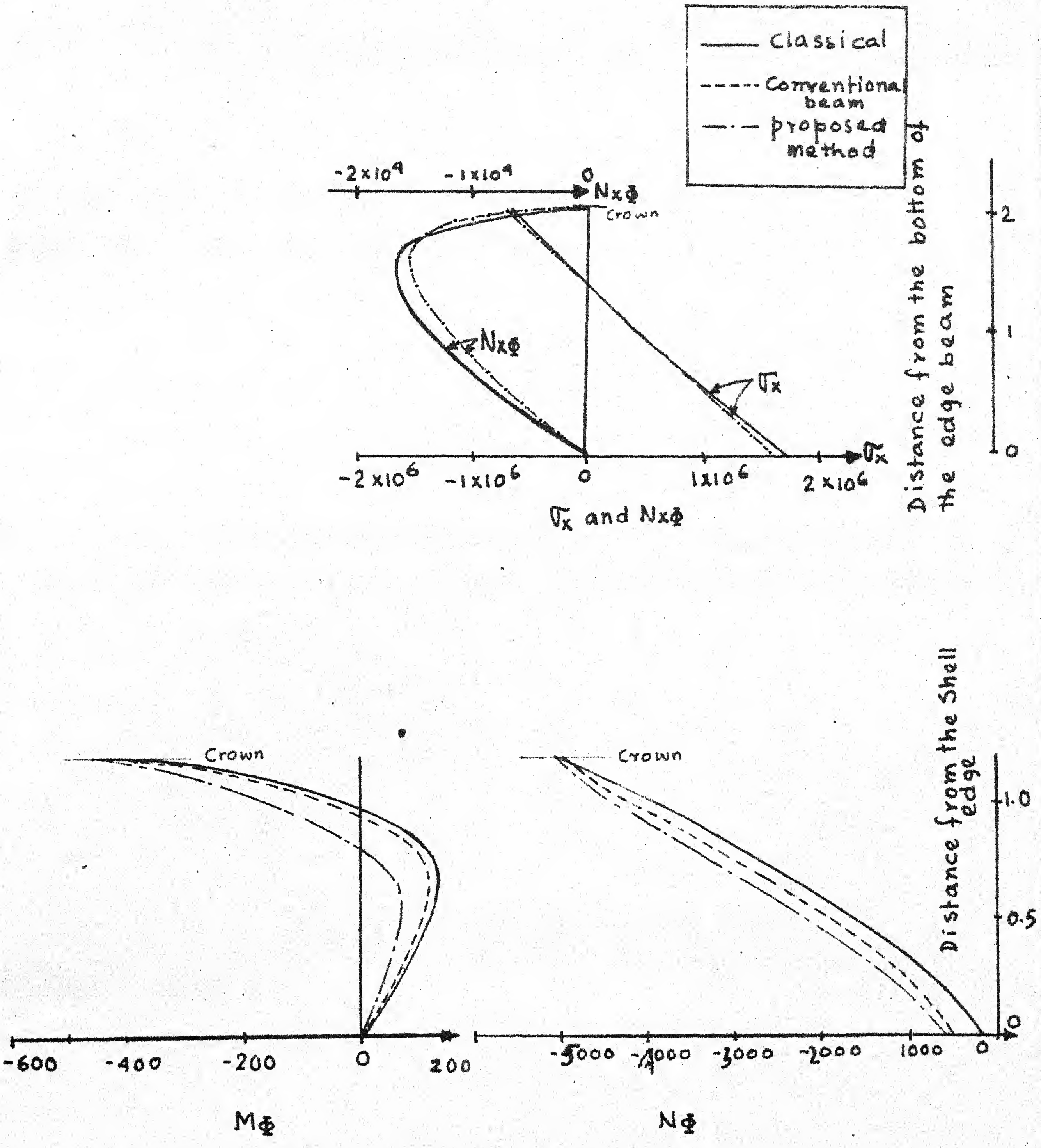


Fig. 4.3. VARIATION OF STRESS RESULTANTS

of a cylindrical shell roof, are $N_{x\phi}$ at x = 0, N_x, N_ϕ and M_ϕ at x = L/2. On the basis of these stress resultants the reinforcements are devided into two groups, 1) Force reinforcements, 2) Moment reinforcements. These are explained below.

1) Forces Reinforcements

i) Longitudinal reinforcements, this is being to overcome N_x.

ii) This is being due to N_ϕ

iii) Diagonal tension created by the combination of N_ϕ, N_x and $N_{x\phi}$

N_x, N_ϕ and $N_{x\phi}$, action on a small element of the shell, may be considered as orthogonal stress in a plane stress system.

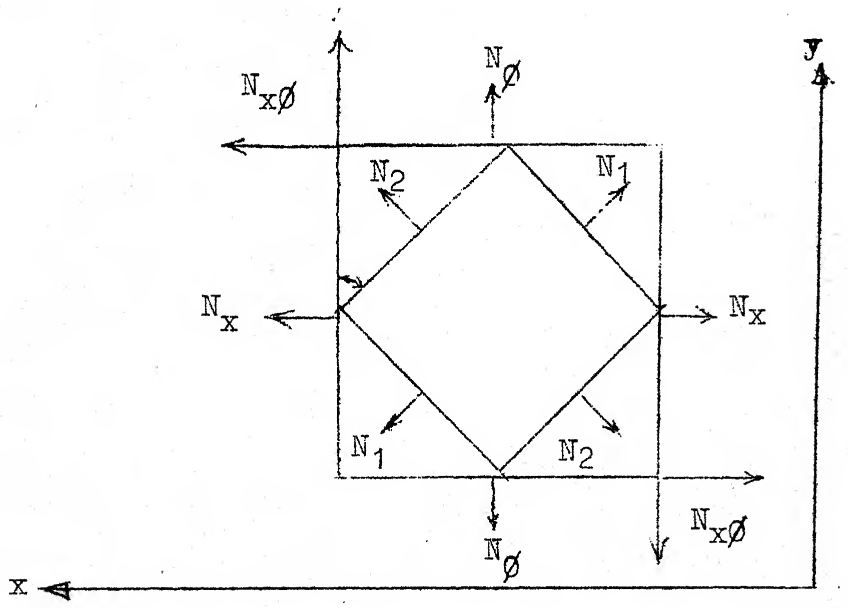


Fig. 4.3 PRINCIPAL STRESSES IN SHELL ELEMENTS

In true theoretical sense the combinationshould be of N_x, N_ϕ, $N'_{x\phi}$, where $N'_{x\phi} = N_{x\phi} - M_{x\phi}/R$. Normally $M_{x\phi}$ is negligible

$$\therefore \quad N'_{x\phi} = N_{x\phi}$$

Principal stresses, $N_{1,2} = \dfrac{N_x + N_\phi}{2} \pm \frac{1}{2} \left\{ (N_x - N_\phi)^2 + 4 N_{x\phi}^2 \right\}^{\frac{1}{2}}$ (4.21 a)

and the direction is given by $\tan 2\theta = \dfrac{2 N_{x\phi}}{N_x - N_\phi}$ (4.21 b)

Thus eqs. of 4.21 provide all the necessary information, to design the reinforcement, these should be examined at x = 0, L/4 and L/2.

At $x = L/2$, $N_{x\phi} = 0$, therefore $N_1 = N_x$ and $N_2 = N_\phi$

where $\theta = 0°$ or $90°$

At $x = L/4$, $N_{1,2} = \dfrac{1}{V_2} \left[\dfrac{N_x - N_\phi}{2} \pm \left\{ (N_x - N_\phi)^2 + 4 N_{x\phi} \right\}^{\frac{1}{2}} \right]$

and $\tan 2\theta = 2 N_{x\phi} / (N_x - N_\phi)$.

At $x = 0$, N_x and N_ϕ are zero. Thus $N_{1,2} = \pm N_{x\phi}$

and $\tan 2\theta = 2 N_{x\phi}/0 \quad \infty \quad \therefore \quad \theta = 45°$.

2) Moment Reinforcements. This is transverse reinforcements to overcome the bending moment M_ϕ.

The analysis of shell has been based on the assumption of homogeneity of the material. In other words, the elastic

20 m.m. ϕ is provided from bottom to shell upto $\phi = 20°$.

ii) For Diagonal tension

Maximum $N_{x\phi}$ = 15380 kg/m. Area of steel required is 10.9 cm^2/m. Provide 12 ϕ @ 10 cm c/c.

iii) For Transverse Force, N_ϕ

N_ϕ is always compressive and maximum value is 5010 kg/m. So no reinforcement is required.

2) Moment Reinforcements

Moment reinforcement is required only for M_ϕ and it charges it's Sign at $\phi = 14°$.

Maximum M_ϕ = -424 Kgm/m. To resists this moment by a singly reinforced concrete section the eff. depth required is 5.49 cm If the eff. cover is kept 2 cm then eff. depth available is 7 cm

$\therefore$ O.K.

Steel required to resists this mount = 4.97 cm^2/m , provide 10 ϕ @ 15 cm c/c.

Distribution steel: 10 ϕ @ 30 cm c/c.

4.9 DISCUSSION

a) The formulations, which have been derived in article 4.4 and 4.5 and presented in 6 steps of calculations are fairly simple enough and can be concieved without the knowledge of shell theory, with the help of the tables given, the computational work is simple and

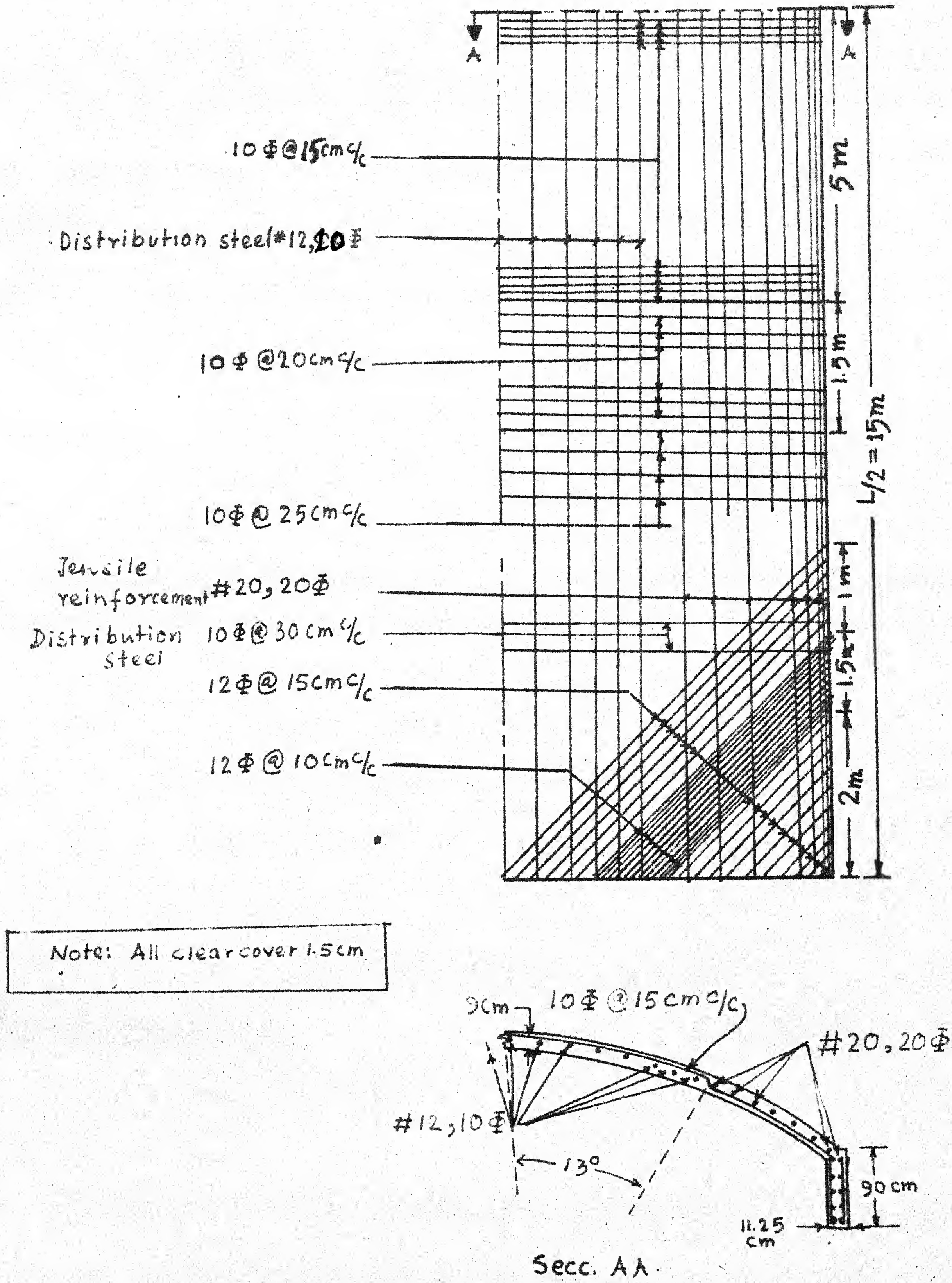


Fig. 4.4 REINFORCEMENT DETAILS (Ex.2)

less tedious. The tables which are given can be used for any values of R/t, L/R and ϕ_k. The single barel, interior and eaterion shell of a multibarrel group can be analysed uniquely Stress resultant can be obtained at any longitudinal section can with $\sin\frac{n\pi x}{l}$ by these formulations. The error involvement with the calculations of arch analysis is less than that which are done by column analogy method or elastic centre method. The error involvement, with the problem of example -1 of article 4.7, for ∇_x at top and bottom are 5.5% (more) and 6.25% (less), for $N_{x\phi}$, M_ϕ and N_ϕ errors are 15% (less), 8.5%(more) and 1.5% (less) respectivly for maximum values

b) The expression, of step 5(d) of the article 4.6 has been derived satisfying the static equilibrium. This could be done by satisfying the vertical deformation compatibility. The vertical deformation of the edge beam at any longitudinal section is given by,

$$w_{vb} = \frac{1}{E_b} \frac{l^4}{bd^3} \left(0.12319\, V - 0.19351 \frac{d}{L} N_{x\phi_4} + \frac{4}{\pi}\, 0.12319\, bd\, \gamma \right) \sin\frac{n\pi x}{l} \qquad (4.21)$$

From the eqn. 4.8 the vertical deformation of the shell is given as (assuming external load varies with Sin mx)

$$w_{vs} = \frac{1}{E_s} \left[(T^3 k_{mv} + T k_{mv})^2 V + (T^3 k_{mvh}) H + (T^3 k_{mvm}) M' + T^3 K_{11} \right] \sin mx \qquad (4.22)$$

To satisfy the vertical deformation compatibility at the shell beam interface w_{vs} should be equal to w_{vb}. Therefore vertical deformation compatibility yields,

$$(T^3 k_{mv} + T k_{nv} - 0.12319\, c_b)\, V + (T^3 k_{mvh}) H + (T^3 k_{mvm}) M' = 1 - T^3 K_{11} + c_b (w_{vbg} - w_{vbs}) \tag{4.23}$$

Where, $c_b = \dfrac{E_b}{E_s}\,\dfrac{l^4}{bd^3}$, $w_{vbg} = \dfrac{4}{\pi}\, 0.12319\, bd\gamma$,

$$w_{vbs} = 0.19351\, N_{x\phi 4}\, \frac{d}{l}$$

The steps 5(d) of article 4.6 will be changed as follow:

Step 5(d) of article 4.6:

$T = R/t$, $c_b = \dfrac{E_b}{E_s}\,\dfrac{l^4}{bd^3}$, $w_{vbg} = \dfrac{4}{\pi}\, 0.12319\, bd$, $w_{vs} = 0.19351\, \dfrac{d}{L}\, N_{x\phi_4}$

$A_{11} = T^3 k_{mv} + T k_{nv} - 0.12319\, c_b$

$A_{12} = A_{21} = T^3 k_{mvh}$

$A_{13} = A_{31} = T^3 k_{mvm}$

$A_{22} = T^3 k_{mh} + T k_{nh}$

$A_{23} = A_{32} = T^3 k_{mhm}$

$A_{33} = T^3 k_{mm}$

$K_{11} = 12 \displaystyle\int_{\phi_k}^{\phi_k} \frac{M_s}{R} \sin \phi_k \, d\phi$ and K_{21}, K_{22} and K_{31} are given in step 5(d). of article 4.6

Above integral may be evaluated by Simpson's rule.

$C_1 = T^3 K_{11} + c_b\, (w_{vbg} - w_{vbs})$

$C_2 = E_s w_h - T^3 K_{21} + K_{22}$

$C_3 = E_s w_m - T^3 K_{31}$

Therefore vertical, horizontal and rotational deformation compatibility at the interface yields,

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} V \\ H \\ M' \end{bmatrix} \quad \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \qquad (4.24)$$

The bouhdery conditions are:

i) For single barrel shell H=0 and $M' = 0$

ii) For interior shell of multibarrel group, $w_h=0$, $w_m=0$ substituting these boundery conditions m eq. 4.2 and solving them V, H and M' could be defined uniquly. The final stress resultants will be obtained from step 6 of article 4.9.

CHAPTER-V

CONCLUSION AND RECOMMENDATION FOR FURTHER WORK

The following conclusions are drawn based on the present investigation.

5.1 EFFECT OF SHELL DIMENSIONS:

1. Reinforced concrete shells: Magnitudes of stress resultants decrease with increase of semicentral angle while increase with span to radius and depth to span ratios by both beam and classical theories. The influence of membrane solution on the total solution increases with semicentral angle. Beam theory agrees closely with classical theory for any values of radius to thickness and depth of edge beam to span ratios when span to radius ratio is greater than 3.. Accuracy of beam theory depends on semicentral angle to a great extent. Higher value of semicentral angle for higher value of L/R ratio, yields the stress resultants with 4 to 10 % error. The preferable range of depth of edge beam to span ratio is 0.03 to 0.05.

Vertical, horizontal and rotational deformations increase with span to radius and depth of edge beam to span ratios while decrease with the increase ofsemicentral angle for classical theory. Beam theory also exhibits the same feature of vertical deformations. Restraints of horizontal

and rotational deformations produce stress resultants which are in a nature to compensate each other. The ratio of the magnitude of vertical deformation, determined by classical theory to that determined by beam theory is in the range of 0.262 to 1.43. Depth of edge beam to span does not have much influence as span to radius ratio and semicentral angle have on this range. The range could be reduced as 0.8 to 1.43 with the proper selection of ϕ_k and L/R ratio.

2. Prestressed Shell: Stress resultants are reduced by prestressing force for suitable depth of edge beam for all values of span to radius ratio and semicentral angle by classical theory. Tension occurs at shell edge for depth to span ratio 0.05 and 0.04. Beam theory does not give any tension for higher values of depth to span ratio. No exact conclusion could be drawn about the influence of semicentral angle and span to radius ratio on the agreement between classical and beam theories in the variation of stress resultants. No shell with depth of edge beam to span ratio $\geqslant$ 0.04 could be designed by beam theory.

Beam theory underestimates the vertical deformation very much. From Table 5.1 it is evident in this respect two theories can not be compared except for a few cases. Horizontal and rotational deformation have higher magnitudes than those of reinforced concrete shell.

Table 5.1 COMPARISON OF BEAM AND CLASSICAL THEORIES

L/R	ϕ_k	d/L	e_o/d	e_m/d	p	ε PSC	ε RCC	δ PSC	δ RCC	Remarks
3.33	30	0.03	0.4	0.4	0.4	31.00	5.5	-0.135	0.89	
	40					53.50	27.75	0.0715	0.388	
	45					61.52	45.82	0.105	0.262	
5.00	30	0.03	0.4	0.4	0.4	30.00	5.7	0.078	1.030	
	40					23.60	10.0	0.14	0.800	
	45					15.00	15.5	0.16	0.590	
10.00	30	0.03	0.4	0.4	0.4	69.60	8.0	0.6	1.08	
	40					69.2	6.2	0.524	1.156	
	45					72.0	11.5	0.458	1.43	
3.33	30	0.05*1	0.4	0.4	0.4	56.5	9.1	0.016	1.510	
		0.04*1				50.0	4.0	-0.123	1.250	
		0.03				31.0	5.55	-0.135	0.89	
		0.02				12.2	11.5	-1.1	0.73	
3.33	30	0.03	0.0	0.0	0.4	5.0	-	-	-	
			0.0	0.4		42.7	-	-	-	
			0.4	0.0		4.5	-	-	-	
			0.4	0.4		31.0	-	-	-	
			0.2	0.4		41.0	-	-	-	
			0.4	0.2*2		40.0	-	-	-	
3.33	30	0.3	0.4	0.4	0.8*3	-25.0	-	0.4	-	
					0.6	57.0	-	0.307	-	
					0.4	31.0	-	-0.135	-	
					0.2	14.0	-	-1.77	-	
					0.0	-	5.5	-	0.89	

ε = Maximum percentage of error in the variation of σ_x by beam theory.

δ = (w_v) beam theory/(w_v) classical theory

PSC and RCC indicate prestressed and reinforced concrete shell respectively.

* 1 Tension occurs at the shell edge by classical theory

2 At the bottom of shell edge beam theory gives compression while classical theory gives tension

3 Beam theory gives tension at crown.

5.2 EFFECT OF PRESTRESSING FORCE:

The reduction of stress resultants depends not only on the prestressing force but the combination of depth of the edge beam end and mid eccentricities. Difference between beam and classical theories decreases with sag. For shells, provided with axially prestressed edge beam, the beam theory agrees with classical theory within an error of 5 to 10 percent.

Prestressing force is the most influencial parameter for the disagreement between beam and classical theories, with suitable combinations of depth of edge beam end and mid eccentricities. N_x and $N_{x\phi}$ decrease with the increase of prestressing force. Beam theory gives the same feature. In classical theory prestressing has a very sensitive influence in the region of edge beam for the distribution of N_x. Its influence in the distribution of $N_{x\phi}$, by classical theory, is not significant while beam theory indicates its significant influence. M_ϕ changes its sign and N_ϕ decreases with the increase of prestressing force by both the theories. Shells with the coefficients of prestressing force $\geqslant 0.4$ could not be designed by beam theory.

The downward vertical deformation for non prestressing shell gradually decreases and ultimately attains upword deformation with the increase of prestressing force for both the theories. Rotational and horizontal deformations

increased with prestressing.

Beam theory assumes loading is uniformly distributed over the surface or chord area of the shell and neglects the relative displacements within each transverse section. These are not correct for shells with edge beam. For reinforced concrete shell the corrective line loads for edge beam are not much while for prestressed shell it has high values. The vertical displacements, estimated by beam theory for reinforced concrete shell is well comparable with that by classical theory while for prestressed shell this is not true. Though beam theory assumes that the rotational and horizontal deformations are negligible, which are not correct, but this assumption does not have much influence on the agreement of beam and classical theories as the restraints against these deformations produces the stresses which are in a nature to compensate each other. The most influential point is the discrepancy in vertical deformation. This discripency increases with the corrective line loads due to edge beam, which inturn directly proportional to the prestressing force. This is the main reason of the disagreement between the shell and beam theory for reasonably higher values of prestressing force.

5.3 PROPOSED METHOD:

The proposed method which is detailed in 6-steps, given article 4.6, has been formulated considering the variation of external load with Sin $\frac{n\pi x}{l}$ in longitudinal direction. The arch analysis portion, developed by strain energy method , is simple and less tedious. The errors in the computations with the help of tables which are given in appendix are small. Maximum number of simultaneous equations to be solved is 3 which is not at all difficult for compution. Shells with any type of boundary conditions can be analysed by this method. The values of the coefficients, given in the tables, are independent of structural geometry of the shell and can be used for any values of span to radius ratio, depth of edge beam to span ratio, thickness to radious ratio and semicentral angle. Stress resultants at any longitudinal section can be determined by this method.

5.4 SUMMARY OF THE CONCLUSIONS:

1. Reinforced concrete shell with any values of R/t, d/L and ϕ_k can be analysed by beam theory with the proposed method for L/R more than 3.

Single barrel shells, exterior and interior of multibarrel shells could be analysed by beam theory with the proposed method.

2. The error varies 4 to 10% for N_x, 2 to 12 % for $N_{x\phi}$. Maximum error occurs for shells having higher values of L/R with lower values of ϕ_k and lower values of ϕ_k with higher values of L/R.

Errors can be minimised with the proper selection of ϕ_k as higher the value of L/R higher the value of ϕ_k.

3. Prestressing has very sensitive influence on N_x in the edge beam while in the shell portion it is not such sensitive. Change of $N_{x\phi}$ with the introduction of pressing is not much significant. Influence of prestressing on N_ϕ and M_ϕ has same level of influence throughout the shell.

4. The reduction of stress resultants is not dependent only on p but on the suitable combination of d/L, e_o/d and e_m/d. along with p.

Shells with axially prestressed edge beam can be analysed by the proposed method of beam theory. Shells provided with d/L $\geqslant$ 0.04 and coefficient of prestressing force more than or equal 0.4 can not be analysed by beam theory.

6. The proposed method to analyse shell with beam theory is systematic, simple and less tedious. Tables, which are given to analyse a shell, can be used for any values of

R/L and R/t and ϕ_k. The stress resultant at any longitudinal section can be estimated by this method.

5.5 SCOPE OF FURTHER WORK:

1. Analysis of cylindrical shells by beam theory with different support and loading condition.

2. It appears worthwhile to extend the analysis for the outer shells for 6 boundary conditions for beam theory and eight boundary conditions for classical theory and to study the effect of higher Fourier terms on the proposed method.

3. Development of the algorithm of to attain the optimal design criterion of cylindrical shells by beam theory.

4. It is worth-while to prepare the design table of prestressed concrete cylindrical shell by beam theory.

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APPENDIX

A.1 SHELL COEFFICIENTS

Table A.1.1 SHELL COEFFICIENTS

(ϕ Measured from the Crown)

$$F = 2\bar{R}\begin{bmatrix} (aB_1 - bB_2)\cos\beta_1\phi\cosh\alpha_1\phi - (aB_2 + bB_1)\sin\beta_1\phi\sinh\alpha_1\phi \\ (cB_3 - dB_4)\cos\beta_1'\phi\cosh\alpha_1'\phi - (cB_4 + dB_3)\sin\beta_1'\phi\sinh\alpha_1'\phi \end{bmatrix}$$

F	R	B_1	B_2	B_3	B_4
M_ϕ	$-\frac{2D}{r^2}\sin kx$	$n^2(1+V)$	n^2	$n^2(V-1)$	n^2
M_x	$2Dk^2\sin kx$	1	0	1	0
Q_x	$-\frac{2Dk^3}{V}\cos kx$	1	1	-1	1
N_ϕ	$\frac{4Drk^4}{V^2}\sin kx$	0	1	0	-1
N_x	$-\frac{4Drk^4}{V^3}\sin kx$	-1	$1+V$	1	$1-V$
Q_x'	$-\frac{2Dk^3}{V}\cos kx$	$V+2$	2	$V-2$	2
u_x	$\frac{4Drk^3}{hEV^3}\cos kx$	-1	$1+V$	1	$1-V$
w_r	$2\sin kx$	1	0	1	0

with $\nu = 0$, so that $D = Et^3/12$

$$k = \frac{n\pi}{l}$$

Table A.1.2 SHELL COEFFICIENTS (Ø Measured from the Crown)

$$F = 2\bar{R} \begin{matrix} (aB_1 - bB_2)\cos\beta_1\phi \sinh\alpha_1\phi - (aB_2 + bB_1)\sin\beta_1\phi\cosh\alpha_1\phi \\ (cB_3 - dB_4)\cos\beta_1'\phi \sinh\alpha_1'\phi - (cB_4 + dB_3)\sin\beta_1'\phi\cosh\alpha_1'\phi \end{matrix}$$

F	R	B_1	B_2	B_3	B_4
$Q\phi$	$-\frac{Dk^3}{(V)^{3/2}}\sin kx$	$m_1 - n_1$	$m_1 + n_1$	$-(m_2 + n_2)$	$m_2 - n_2$
Q_ϕ'	$-\frac{2Dk^3}{(V)^{3/2}}\sin kx$	$m_1(1-V) - n_1$	$m_1 + n_1(1-V)$	$-m_2(1+V) - n_2$	$m_2 - n_2(1+V)$
$N_{x\phi}$	$\frac{4Drk^4}{(V)^{5/2}}\cos kx$	$-n_1$	m_1	n_2	$-m_2$
u_y	$\frac{4Drk^3}{Eh(V)^{7/2}}$	$m_1 + n_1(1-V)$	$n_1 - m_1(1-V)$	$-m_2 + n_2(1+V)$	$-n_2 - m_2(1+V)$
θ^1	$\sin kx$	$\frac{2\alpha_1}{r} + \frac{(\bar{R}.B_1)_v}{r}$	$\frac{2\beta_1}{r} + \frac{(\bar{R}.B_2)_v}{r}$	$\frac{2\alpha_1'}{r} + \frac{(\bar{R}.B_3)_v}{r}$	$\frac{2\beta_1'}{r} + \frac{(\bar{R}.B_4)_v}{r}$
$M_{x\phi}$	$\frac{2Dk}{r}\cos kx$	α_1	β_1	$\beta\ \alpha_1'$	β_1'

$\nu = 0$, so that $D = Eh^3/12$

1 Observe that one part of the coefficients B_1 etc., for θ are obtained from v, that is ,

$$(\bar{R}.B_1)_v/r = \frac{4Dk^3}{Eh(V)^{7/2}} m_1 + n_1(1 - V)$$

The subscript v indicates u_y

A.2 COEFFICIENTS OF THE TRANSVERSE SECTIONAL PROPERTIES

Table A.2.1 COEFFICIENTS FOR C.G.

Ø (in degree)	Ø (in rad)	Cos Ø	Sin Ø	k_z
45.00	0.78540	0.70711	0.70711	0.09968
42.50	0.74176	0.73728	0.67559	0.08921
40.00	0.69813	0.76604	0.64279	0.07927
37.50	0.65450	0.79335	0.60876	0.06988
35.00	0.61087	0.81915	0.57358	0.06104
32.50	0.56723	0.84339	0.53730	0.05277
30.00	0.52360	0.86603	0.50000	0.04507
27.50	0.47997	0.88701	0.46175	0.03795
25.00	0.43633	0.90631	0.42262	0.03143
22.50	0.39270	0.92388	0.38268	0.02550
20.00	0.34907	0.93969	0.34202	0.02018
17.50	0.30543	0.95372	0.30071	0.01548
15.00	0.26180	0.96593	0.25882	0.01138
12.50	0.21817	0.97630	0.21644	0.00791
10.00	0.17453	0.98481	0.17365	0.00507
7.50	0.13090	0.99144	0.13053	0.00285
5.00	0.08727	0.99619	0.08716	0.00127
2.50	0.04363	0.99905	0.04362	0.00032
-0.00	-0.00000	1.00000	-0.00000	-0.00000

Table A.2.2 COEFFICIENTS FOR MOMENT OF INERTIA
(ϕ Measured from crown)

ϕ_k^*	k_I	ϕ_k^*	k_I	ϕ_k^*	k_I	ϕ_k^*	k_I
45.00	0.01216	44.50	0.01152	44.00	0.01091	43.50	0.01032
43.00	0.00976	42.50	0.00922	42.00	0.00871	41.50	0.00822
41.00	0.00775	40.50	0.00730	40.00	0.00687	39.50	0.00647
39.00	0.00608	38.50	0.00571	38.00	0.00536	37.50	0.00502
37.00	0.00470	36.50	0.00440	36.00	0.00411	36.50	0.00384
35.00	0.00358	34.50	0.00334	34.00	0.00311	33.50	0.00289
33.00	0.00269	32.50	0.00249	32.00	0.00231	31.50	0.00214
31.00	0.00198	30.50	0.00182	30.00	0.00168	29.50	0.00155

* ϕ_k , given in degrees

Table A.3.1 COEFFICIENTS FOR ARCH CALCULATIONS

ϕ_k	k_{mv}	k_{mh}	k_{mm}	k_{mvh}	k_{mvm}	k_{mhm}	k_{nv}	k_{nh}
45.00	12.84956	0.84956	18.84956	-2.57522	13.32865	-3.64191	0.28540	1.28540
44.50	12.47837	0.80552	18.64012	-2.47193	13.06503	-3.52675	0.27675	1.27680
44.00	12.11272	0.76325	18.43068	-2.37148	12.83302	-3.41388	0.26825	1.26764
43.50	11.75265	0. 72272	18.22124	-2.27384	12.54267	-3.30329	0.25990	1.25853
43.00	11.39819	0.68386	18.01180	-2.17896	12.28402	-3.19497	0.25171	1.24927
42.50	11.04940	0.64664	17.80236	-2.08682	12.02710	-3.08889	0.24367	0.123986
42.00	10.70631	0.61100	17.59292	-1.99739	11.77196	-2.98505	0.23578	1.23030
41.50	10.36895	0.57690	17.38348	-1.91062	11.51864	-2.88342	0.22804	1.22058
41.00	10.03735	0.54430	17.17404	-1.82647	11.26718	-2.78400	0.22045	1.21072
40.50	9.71155	0.51313	16.96460	-1.74492	11.01763	-2.68677	0.21301	1.20070
40.00	9.39156	0.48337	16.75516	-1.66592	10.77001	-2.59170	0.20573	1.19054
39.50	9.07742	0.45497	16.54572	-1.58943	10.52437	-2.49879	0.19859	1.18022
39.00	8.76914	0.42788	16.33628	-1.51541	10.28075	-2.40801	0.19160	1.16975
38.50	8.46675	0.40206	16.12684	-1.44383	10.03919	-2.31935	0.18477	1.15914
38.00	8.17024	0.37746	15.91640	-1.37464	9.79973	-2.23279	0.17808	1.14837
37.50	7.87965	0.35460	15.70796	-1.30781	9.56240	-2.14831	0.17154	1.13746

Contd.....

ϕ_k	k_{mv}	k_{mh}	k_{mm}	k_{mvh}	k_{mvm}	k_{mhm}	k_{nv}	k_{nh}
37.00	7.59497	0.33180	15.49852	-1.24328	9.32724	-2.06582	0.16514	1.12640
36.50	7.31621	0.31065	15.28908	-1.18103	9.09429	-1.98551	0.15889	1.11520
36.00	7.04337	0.29056	15.07964	-1.12100	8.86359	-1.90716	0.15279	1.10385
35.50	6.77646	0.27150	14.87020	-1.06315	8.63517	-1.83081	0.14683	1.09235
35.00	6.51547	0.25344	14.66076	-1.00745	8.40907	-1.75644	0.14102	1.08071
34.50	6.26040	0.23632	14.45132	-0.95385	8.18532	-1.68403	0.13535	1.06893
34.00	6.01123	0.22013	14.24189	-0.90230	7.96396	-1.61357	0.12982	1.05700
33.50	5.76796	0.20482	14.03245	-0.85276	7.74502	-1.54503	0.12443	1.04494
33.00	5.53057	0.19035	13.82301	-0.80519	7.52855	-1.47839	0.11919	1.03273
32.50	5.29905	0.17670	13.71357	-0.79554	7.31456	-1.41365	0.11408	1.02039
32.00	5.07337	0.16383	13.40413	-0.71577	7.10310	-1.35072	0.10911	1.00790
31.50	4.85352	0.15170	13.19469	-0.67384	6.89420	-1.28964	0.10428	0.99528
31.00	4.63946	0.14030	12.98525	-0.63369	6.68790	-1.23038	0.09958	0.98253
30.50	4.43117	0.12957	12.77581	-0.59530	6.48421	-1.17291	0.09502	0.96964
30.00	4.22862	0.11951	12.56637	-0.55860	6.28381	-1.11720	0.09059	0.95661
29.50	4.03178	0.11006	12.35693	-0.52356	6.08484	-1.06324	0.08629	0.94346

A. 4 COEFFICIENTS FOR STRESS RESULTANTS

Table A.4.1 COEFFICIENTS FOR STRESS RESULTANTS(N_ϕ)

ϕ_k	c_{vn}	c_{hn}	ϕ_k	c_{vn}	c_{hn}
45.00	-0.70711	-0.70711	42.50	-0.67559	-0.73728
40.00	-0.64279	-0.76604	37.50	-0.60876	-0.79335
35.00	-0.57358	-0.81915	32.50	-0.53730	-0.84339
30.00	-0.50000	-0.86602	27.50	-0.46175	-0.88701
25.00	-0.42262	-0.90631	22.50	-0.38268	-0.92388
20.00	-0.34202	-0.93969	17.50	-0.30071	-0.95372
15.00	-0.25882	-0.96593	12.50	-0.21644	-0.97630
10.00	-0.17365	-0.98481	7.50	-0.13053	-0.99144
5.00	-0.08716	-0.99619	2.50	-0.04362	-0.99905
0.00	-0.00000	-1.00000			

Table A.4.2 COEFFICIENTS OF STRESS RESULTANTS(M_ϕ)

ϕ	c_{vm}	c_{hm}	ϕ	c_{vm}	c_{hm}
ϕ_k = 30.00			ϕ_k=32.50		
30.00	0.00000	0.00000	32.50	0.00000	0.00000
25.00	0.07738	-0.04028	30.00	0.03730	-0.02263
20.00	0.15798	-0.07367	25.00	0.11468	-0.06292
15.00	0.24118	-0.09990	20.00	0.19528	-0.09630
10.00	0.32635	-0.11878	15.00	0.27848	-0.12253
5.00	0.41284	-0.13017	10.00	0.36365	-0.14142
0.00	0.50000	-0.13397	5.00	0.45140	-0.15280
			0.00	0.53730	-0.15661

Contd.......

ϕ	c_{vm}	c_{hm}	ϕ	c_{vm}	c_{hm}
ϕ_k = 35.00			ϕ_k=37.50		
35.00	0.00000	0.00000	37.50	0.00000	0.00000
30.00	0.73580	-0.04687	35.00	0.03518	-0.02580
25.00	0.15096	-0.08716	30.00	0.10876	-0.07267
20.00	0.23156	-0.12054	25.00	0.18614	-0.11295
15.00	0.31476	-0.14677	20.00	0.26674	-0.14634
10.00	0.39993	-0.16566	15.00	0.34994	-0.17227
5.00	0.48642	-0.17704	10.00	0.43511	-0.19145
0.00	0.57358	-0.18035	5.00	0.52161	-0.20284
			0.00	0.60876	-0.20665
ϕ_k = 40.00			ϕ_k=42.50		
40.00	0.00000	0.00000	42.50	0.0000	0.00000
35.00	0.06921	-0.05311	40.00	0.03280	-0.02877
30.00	0.14279	-0.09998	35.00	0.10201	-0.08187
25.00	0.22017	-0.14026	30.00	0.17559	-0.12875
20.00	0.30077	-0.17365	25.00	0.25297	-0.16903
15.00	0.38397	-0.19988	20.00	0.33357	-0.20242
10.00	0.46914	-0.21876	15.00	0.41677	-0.22865
5.00	0.55563	-0.23015	10.00	0.50194	-0.24753
0.00	0.64279	-0.23396	5.00	0.58843	-0.25892
			0.00	0.67559	-0.26272
			ϕ_k=45.00		
45.00	0.00000	0.00000	20.00	0.36509	-0.23259
40.00	0.06432	-0.05894	15.00	0.44829	-0.25882
35.00	0.13353	-0.11205	10.00	0.53346	-0.27770
30.00	0.20711	-0.15892	5.00	0.61995	-0.28909
25.00	0.28449	-0.19920	0.00	0.70711	-0.29289